FORECASTING RESEARCH & DEVELOPMENT PROGRAM BUDGETS USING THE WEIBULL MODEL

THESIS

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Abstract

Norden (1970) uses the Rayleigh, which is a degenerative of the Weibull, to model manpower on research and development (R&D) programs. Several research efforts extend his work including Lee, Hogue, and Gallagher (1997) who build R&D program budgets based on Rayleigh expenditures. We demonstrate the theoretical limitations to the Rayleigh model and present the Weibull model, which mitigates those limitations. Using 102 completed R&D defense programs, we develop regression models to predict the requisite shape and scale parameters to forecast Weibull-based budgets. Using the remaining 26 completed R&D programs to validate the robustness of our regression models, we show that 100 and 96 percent of the least squares estimated shape and scale values respectively, fall within a 95 percent prediction interval. We determine the Weibull model's budget projection capability by comparing forecasted Weibull-based budgets to 128 completed R&D program budgets and report an average correlation of 0.607. To determine the significance of our results we compare forecasted Rayleigh-based budgets to the same 128 completed program budgets. Using the Weibull over the Rayleigh model when applying Lee, Hogue, and Gallagher's (1997) methodology, we improve initial budget profile projections on average 60 percent.

FORECASTING RESEARCH & DEVELOPMENT PROGRAM BUDGETS USING THE WEIBULL MODEL

I. Introduction to the Research

Problem Statement

When Military Departments plan Research and Development (R&D) programs, they must project a budget profile. Lee, Hogue, and Gallagher (1997) present a method to derive an appropriate budget for a given expenditure profile. Porter (2001) and Unger (2001) indicate that a Weibull distribution model fits R&D program expenditures well. When a program begins; however, no method currently exists to determine the appropriate Weibull shape and scale parameters. This research provides a means to determine initial R&D program budgets based on a Weibull model.

General Issue

The program manager must project a budget profile when a new R&D program is planned. However, past cost and schedule projections for most new start R&D programs significantly underestimate the final program cost and schedule. Past research shows that R&D programs historically require significant increases to projected costs and program schedule duration. Unger (2001) studied 37 historical defense R&D programs and found that the final budgets and actual program durations exceed the initial budgets and estimated program durations on an average of 20 and 25 percent respectively. In some

cases the program cost and schedule growth is very extensive. For example, Gallagher and Lee (1996) report that the R&D effort for the NavStar Global System cost nearly three times the original estimate, and the Trident Submarine program schedule stretched from 5 to 16 years, an increase of 320 percent. The Military Departments and the Office of the Secretary of Defense (OSD) Program Analysis and Evaluation (PA&E) need an accurate way to determine the reasonableness of new start R&D budget projections.

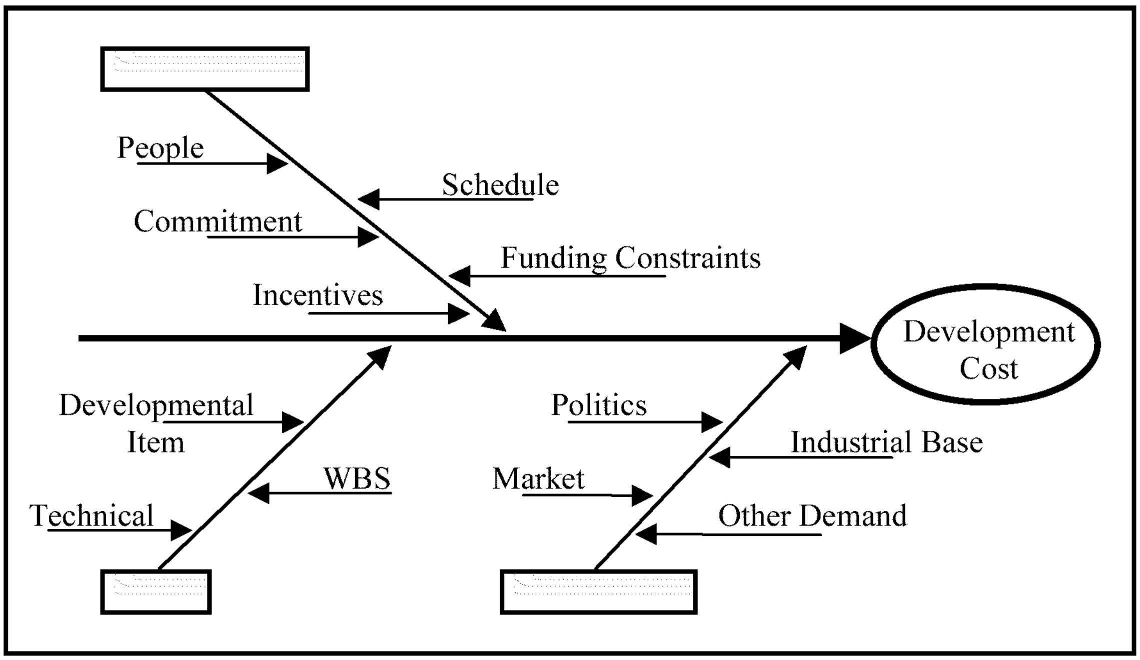

Figure 1. Factors Contributing to Development Cost (Belcher and Dukovich, 1999)

Initial program development costs, schedules, and budget profiles are difficult to project considering the number of possible factors that affect the estimates. Based on expert opinion, Belcher and Dukovich (1999) present a macro view of proposed contributors to how development programs incur costs. Figure 1 shows their 3 major and 12 supporting contributors they propose. Unger's (2001) study shows that 1 of the 12 factors (funding constraints) explains 53.4 percent of cost overrun and 50.5 percent of

schedule slip variation. Our study focuses on developing a method for determining an appropriate budget for new R&D program starts that should reduce cost and schedule growth.

Lee, Hogue, and Gallagher (1997) present a method to derive appropriate budgets by using a Rayleigh distribution to project an expenditure profile. Unger (2001) evaluates 37 final R&D program expenditure profiles and finds that only 52 percent fit a Rayleigh distribution. Porter (2001) and Unger (2001) show that R&D program budgets more often support a Weibull distribution of expenditures. The Rayleigh distribution is a special case of the Weibull distribution. Since the Weibull is more flexible than the Rayleigh, we expect that a higher percentage of expenditure profiles from R&D programs fit a Weibull distribution.

Research Approach

Lee, Hogue, and Gallagher (1997) present a least squares approach that accounts for spend out patterns to develop annual budgets from a point estimate. They use the Rayleigh distribution with predicted total program cost and desired duration to model expenditures. Because the Rayleigh distribution suffers from theoretical limitations, we model R&D program expenditures with a Weibull distribution to mitigate those limitations. However, no method currently exists for determining the Weibull shape and scale parameters at program inception. Using multiple regression, our research determines a mathematical relationship between the Weibull shape and scale parameters with predictors including lead service, type of program, program final cost and duration.

While we use actual final cost and duration to develop this approach, program managers will have to rely on estimates.

The model building data for the Weibull shape and scale parameters are determined by fitting a Weibull distribution to derived program expenditures. The expenditure profiles are derived from 128 completed R&D program budgets from the Selected Acquisition Report (SAR) by applying average service spend out rates. We test our assumption that R&D program expenditures fit a Weibull model using Komolgorov-Smirnov, Cramer-von Mises, and Anderson-Darling goodness-of-fit statistics. The study finds a relationship between the Weibull shape and scale parameters and predictors from the SAR. Our method predicts the requisite shape and scale parameters for the Weibull model to provide a better quantitative method to determine initial R&D program budget profiles.

<u>Scope</u>

Several factors contribute to R&D program cost overrun and schedule slip. This research narrows the scope of contributing factors to funding constraints attributed to budget profiles that inadequately meet fiscal expenditure requirements. We seek to minimize R&D program cost and schedule growth by developing a better quantitative approach to forecast initial program budget profiles. Past research indicates that R&D budgets result in expenditures that follow a Weibull distribution. This research seeks to find relationships between estimated Weibull parameters for completed programs and explanatory variables including service type, program type, program final cost and duration. These relationships may be used to determine an expenditure distribution based

on estimated total cost and desired program duration. An analyst may determine a budget from Weibull-based expenditures using Lee, Hogue, and Gallagher's (1997) approach.

Research Benefits

The research seeks to minimize R&D program cost and schedule growth by developing a better quantitative approach to project initial program budget profiles. This quantitative technique is potentially useful to all Military Departments and OSD PA&E to verify the reasonableness of proposed R&D program budget profiles.

Chapter Summary

This chapter proposes the Weibull model as a better approach to forecasting R&D program budget profiles. Chapter Two explores prior research that uses the Rayleigh and Weibull models. We discuss the limitations to the Rayleigh model and present the Weibull model, which mitigates those limitations. Chapter Three explains the methodology applied to historical DoD program data to predict the requisite shape and scale parameters to forecast Weibull-based R&D budgets. Chapter Four provides validation results to the methodology applied to historical DoD program data outlined in Chapter Three. We include an average correlation comparison between Rayleigh-based and Weibull-based budget profile projections to 128 completed R&D budget profiles. Chapter Five summarizes and concludes our proposed methodology, while addressing possible limitations and future research.

II. Literature Review

Chapter Overview

Developing a budget profile that meets R&D expenditure requirements is essential to the success of a new program. As stated in Chapter One, Unger (2001) finds that over 50 percent of cost overruns and schedule slips are due to funding constraints. Insufficient program funding hinders progress, while over-funded programs inefficiently use resources that could be used elsewhere. Several mathematical models are employed to forecast an appropriate budget profile. Porter (2001) and Unger (2001) review the Beta, Sech-Squared, and Rayleigh Models. Currently, we know of no method to estimate the parameters at program inception for the Beta or Sech-Squared Models.

This chapter relates budgets to expenditures, presents Norden's theory on Rayleigh and summarizes several applications. We present the Rayleigh model, identify Rayleigh model theoretical limitations, and discuss Porter (2001) and Unger's (2001) findings that the Weibull model better fits R&D program expenditures. We apply Norden's theory in terms of the Weibull function and present and graph the Weibull cumulative distribution function and probability density function. This chapter concludes with a discussion of the necessary shape and scale parameters to forecast Weibull-based budget profiles.

When Military Departments plan R&D programs, they must project a budget profile. The defense R&D budget for any given year expends or outlays over several years. For the program to succeed, the budget profile must meet each fiscal year expenditure requirement. The challenge is to ensure sufficient funding in a particular

year given each budget year's outlay pattern. Several models aid in forecasting fiscal requirements, we present the Rayleigh model with its limitations and the Weibull model, which mitigates those limitations. Both models provide an approach for modeling R&D program expenditures in constant dollars over time. Each of the models estimates expenditures by applying a cost factor in constant-year dollars to their respective cumulative distribution functions. Therefore, we present an initial discussion of converting budget profiles to expenditure profiles.

Relating Budget Profiles to Expenditure Profiles

When DoD budgets are determined, multi-year appropriations and inflation affect the computation. The budget or total obligation authority is the necessary funding to execute the program over multiple years. The OSD comptroller prescribes standard expenditure patterns or percentages of R&D funds to be spent in a given year. Outlay rates are the percent spent for each particular budget year. Inflation indices provide the necessary factor to calculate budget requirements in the out years of R&D programs. We convert a budget profile in current dollars to an expenditure profile in constant dollars by applying the outlay rates and removing inflation.

We first apply the outlay rates to the budget profile to obtain the expenditure profile, O_i in current dollars for each ith year with

$$O_i = B_i s_1 + B_{i-1} s_2 + B_{i-2} s_3 + \ldots B_{i-J} s_J, \qquad (1)$$

where B_i is the budget authority for the ith year in current-year dollars, s_j are the outlay rates for the jth year of the budget, where the sum of $s_j \leq 1$, and J is the total number of

years in the expenditure profile. The result is the expenditure profile in current-year dollars, which are yearly expenditures with the affects of inflation.

We then convert the expenditure profile in current-year dollars to constant-year dollars by removing the inflation component with

$$\tilde{O}_i = \frac{O_i}{c_i} \tag{2}$$

where $\tilde{O}_i$ are the expenditures for the *i*th year in constant-year dollars, O_i are the expenditures for the *i*th year in current-year dollars, and c_i is the inflation index for the appropriate *i*th year. The current dollar budget profile is now an expenditure profile in constant-year dollars using formulas (1) and (2). The expenditure profile in constant-year dollars is required to utilize the Rayleigh and Weibull models that theoretically account for development effort, which is not affected by inflation. The next section presents Norden's theory on Rayleigh, summarizes numerous applications and presents and graphs the Rayleigh model.

Norden's Theory on Rayleigh

Norden (1970) applies the Rayleigh distribution to manpower buildup and phase-out on development efforts. Norden bases his theory on skill level increasing linear with time. Because skill level is increasing linear with respect to time, the rate of learning is constant. The rate at which R&D programs acquire skills is not affected by inflation, thus the Rayleigh distribution models constant-year expenditures. The initial ramp-up in the Rayleigh model is due to linear skills acquisition. Since the rate of work completed is

proportional to the work remaining, an exhaustion of the work causes the exponential decrease in the Rayleigh tail (Jarvis and Pohl, 1999:13).

Putnam (1978) applies the Rayleigh model to software development. Based on budgetary data for 50 Computer System Command systems, Putnam determines that the Rayleigh estimates software development well. Since many software programs tend to follow the characteristic growth to a peak and exponential fall-off of the Rayleigh, Putnam declares that the Raleigh model relates to software development.

Later research broadens the Rayleigh model application to DoD contracts. Watkins (1982) applies the Rayleigh model to defense acquisition cost and schedule data on 30 DoD contracts. Watkins reports that the Rayleigh modeled the earned value of contracts well. Abernethy (1984) applies the Rayleigh model to 21 completed Navy contracts. Although Abernethy did not meet his objective to use the Rayleigh model as a forecasting tool, he finds the Rayleigh adequately modeled DoD contract data. Lee, Hogue, and Gallagher (1997) continue the application by demonstrating that the Rayleigh model fits defense R&D acquisition program expenditures in constant-year dollars. Past research substantiates that the Rayleigh CDF models cumulative constant-dollar R&D defense program expenditures well (Gallagher and Lee, 1996:52).

<u>Norden's Theory in Terms of the Weibull</u>

The Rayleigh model is prevalent in R&D expenditure modeling (Norden, 1970; Putnam, 1978; Lee, 1997; Gallagher, 1996). Norden (1970) develops the Rayleigh model based upon engineers solving a fixed number of development project problems with increased linear learning. Although he states linear, Norden notes experiments with other

than linear rates, which leads to the general class of the Weibull models. Porter (2001) and Unger (2001) show that the Weibull function models R&D program expenditures at a higher degree of accuracy than the Rayleigh model.

<u>The Weibull and Rayleigh are Related</u>. The Rayleigh function is a degenerative of the Weibull function. Hines and Montgomery (1980) present the Weibull cumulative distribution function (CDF) with the location parameter as

$$F(t) = 1 - e^{-\left(\frac{t-\gamma}{\delta}\right)^{\beta}}, \tag{3}$$

and the Weibull probability density function (PDF), the derivative of the Weibull CDF, as

$$f(t) = \frac{\beta}{\delta}\left(\frac{t-\gamma}{\delta}\right)^{\beta-1} e^{-\left(\frac{t-\gamma}{\delta}\right)^{\beta}}, \tag{4}$$

where t is time in years, γ is the location parameter, δ is the scale parameter, and β is the shape parameter. The results of the Weibull CDF with (3) always fall between 0 and 1. The cost factor d scales the Weibull CDF to reflect the cumulative program expenditures. We present the Weibull cost model as the CDF in (3) multiplied by a cost factor d with

$$E(t) = d\left(1 - e^{-\left(\frac{t-\gamma}{\delta}\right)^{\beta}}\right). \tag{5}$$

The Weibull cost model calculates a program's total cumulative expenditures at a specified time *t,* for a given shape, scale, and location parameter. The Rayleigh CDF is a special case of the Weibull CDF in (3) when fixing the shape and location parameters to $\beta = 2$ and $\gamma = 0$. We define the Rayleigh CDF as

$$F(t) = 1 - e^{-\left(\frac{t}{\delta}\right)^2}, \tag{6}$$

and the Rayleigh PDF as

$$f(t) = 2\delta^{-2} t e^{-\delta^{-2} t^2}. \tag{7}$$

The foundational concept for the Weibull, and hence the Rayleigh, cost models is based upon solving a fixed number of development project problems with linear learning. The rate at which work is completed is a function of performance and remaining work,

$$\frac{dw(t)}{dt} = p(t)[1 - w(t)],$$

where $p(t)$ is the performance and $w(t)$ is the remaining work. Let $z(t) = 1 - w(t)$, so $\frac{dz(t)}{dt} = \frac{-dw(t)}{dt}$ and $\frac{dz(t)}{dt} = -p(t)z(t)$ we solve for $w(t)$. By integrating $\ln(z(t)) = -\int_{\tau=0}^{\tau=t} p(\tau)d\tau$ and evaluating both sides to the power of the base of the natural logarithm, we obtain $z(t) = e^{-\int_{\tau=0}^{\tau=t} p(\tau)d\tau}$. We show the total work completed by substituting back in percent of work remaining at time t, $w(t)$, in the following form

$$w(t) = 1 - e^{-\int_{\tau=0}^{\tau=t} p(\tau)d(\tau)}.$$

We define performance on the program for any given time as a constant k multiplied by time to a constant power, $p(t) = kt^b$. Since $\int_{\tau=0}^{\tau=t} p(\tau)d\tau = \int_{\tau=0}^{\tau=t} k\tau^b d\tau = \frac{k}{b+1}t^{b+1}$,

$$w(t) = 1 - e^{\frac{-k}{b+1}t^{b+1}}.$$

We obtain the Weibull CDF in (3) when $\beta = b+1$ and $\delta = \sqrt[b+1]{b+1/k}$. We obtain the Rayleigh CDF with linear growth in performance over time when the location parameter $\gamma = 0$, $\beta = 2$, and $\delta = \sqrt{2/k}$. The percent work complete according to the Weibull CDF is derived if performance improves over time to a power other than one.

The Rayleigh CDF. Figure 2 presents the Rayleigh CDF. We apply formula (6) given $\delta = 6$ and $\beta = 2$. This figure shows the cumulative percent of expenditures incurred for an R&D effort.

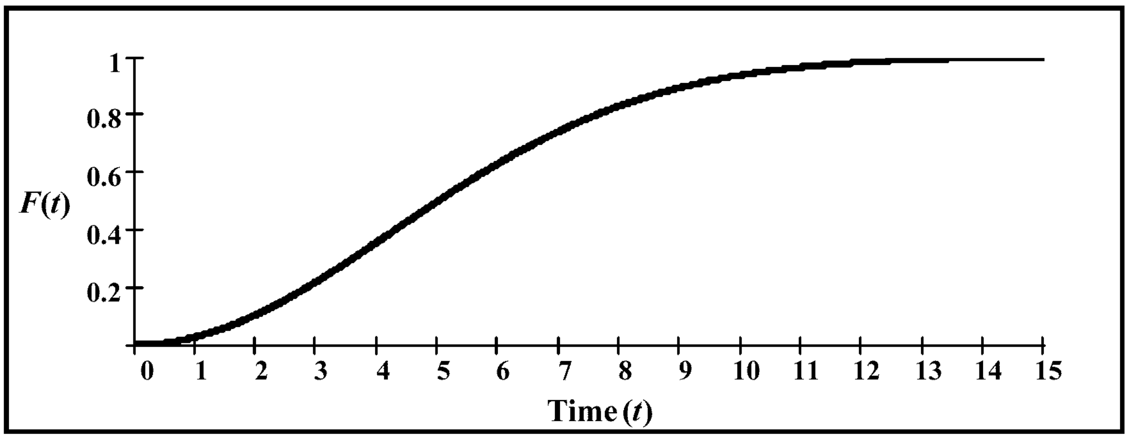

Figure 2. Rayleigh Cumulative Distribution Function (CDF)

The Rayleigh PDF. The derivative of (6) gives the Rayleigh PDF in (7). Figure 3 shows the Rayleigh PDF with the same parameters as in Figure 2. The Rayleigh PDF is the instantaneous rate that the percentage funds are expended.

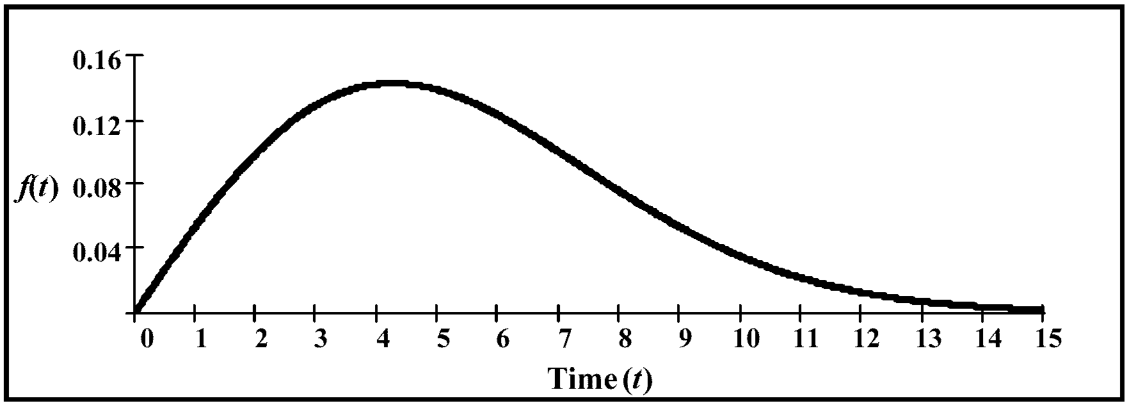

Figure 3. Rayleigh Probability Density Function (PDF)

The Rayleigh Model Parameters. The two varying parameters in the Rayleigh function are the scale and cost factor. The scale parameter, δ, determines the steepness of the Rayleigh CDF curve, and the cost factor, *d,* scales the Rayleigh CDF to reflect the cumulative program expenditures. The δ parameter determines the time period that manpower utilization is maximized (Norden, 1970:126). The results of the Rayleigh CDF with (6) always fall between 0 and 1. We model the cumulative expenditures with

$$E(t) = d\left[1 - e^{-\left(\frac{t}{\delta}\right)^2}\right], \tag{8}$$

where *d* is the cost factor of the R&D project and *t* is the time from program inception to completion (Lee, Hogue, and Gallagher, 1997:31). Figure 4 shows how we apply (8) to illustrate the increasing gradient of the Rayleigh CDF scaled with *d* as the δ parameter decreases.

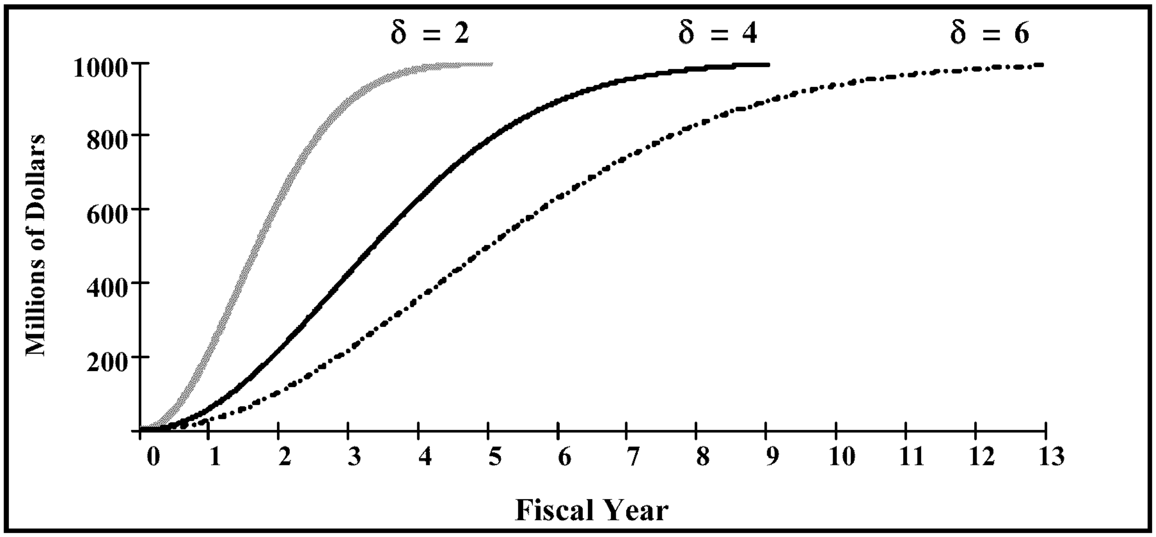

Figure 4. The Rayleigh CDF while changing the δ parameter with $d = 1000$

The derivative of the Rayleigh cost model (8) is the scaled Rayleigh PDF or expenditure rate for a modeled program. We define the Rayleigh PDF scaled with the d parameter as

$$E'(t) = 2\delta^{-2} dte^{-\delta^{-2}t^2}. \qquad (9)$$

Figure 5 demonstrates the effects of changing the δ parameter on the Rayleigh PDF. Notice that as the δ parameter decreases, the time of peak expenditures occurs earlier and higher in the program. The maturity of a program is sooner when the rate of expenditures is higher than another program of equal value (in constant dollars).

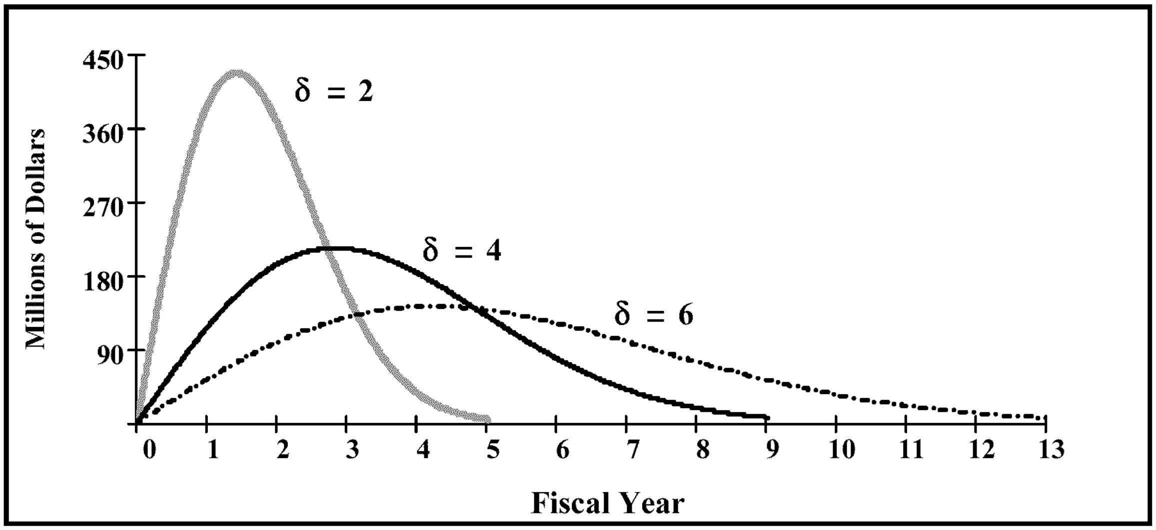

Figure 5. The Rayleigh PDF while changing the δ parameter with $d = 1000$

Figure 5 illustrates how the Rayleigh expenditure distribution provides important information about a modeled program. The shape of the expenditure distribution determines the peak time and magnitude of expenditures and the estimated program duration. Lee, Hogue, and Gallagher (1997) present two methods to determine the appropriate δ parameter. The first technique is based on the time of peak expenditure rate and the other is based on the estimated program completion time.

<u>The Time of Peak Expenditure Rate Technique</u>. This technique uses a program's estimated time of peak expenditure rate to determine the δ parameter. The time of peak expenditure rate is estimated with some degree of reliability. For instance, aircraft R&D program's peak expenditure rate typically occurs at the time of first test flight (Lee, Hogue, and Gallagher, 1997:32). If the peak expenditure rate is known then the δ parameter is defined as

$$\delta = \sqrt{2}t_{peak} \tag{10}$$

by setting (7) equal to zero (Lee, Hogue, and Gallagher, 1997:32). Setting (7) equal to 0 and solving for t results in the peak of a program's expenditure rate with respect to time.

<u>The Estimated Program Completion Time</u>. When the time of peak expenditure rate is unknown, we use the estimated program completion time to determine an appropriate δ parameter. Because the right side of the Rayleigh distribution has an infinite tail, a defined point must determine program completion. Lee, Hogue, and Gallagher (1997) define this point as t_{final}, the time of final development. Final development occurs when the Rayleigh CDF reaches 97 percent of the constant-dollar cumulative R&D expenditures for a project in the Engineering, Manufacturing, and Development phase according to Lee, Hogue, and Gallagher (1997). Lee, Hogue, and Gallagher define the equation as

$$D = E(t_{final}) = 0.97d, \qquad (11)$$

where D is the total R&D estimated program cost in constant dollars and d is the parameter that scales the Rayleigh CDF to program cost. When the program completion time is estimated, then Lee, Hogue, and Gallagher (1997) define the δ parameter as

$$\delta = 0.5345 t_{final}. \qquad (12)$$

In conclusion, either the time of peak expenditure rate or the estimated program completion time technique determines an appropriate δ parameter for the Rayleigh model. The time scale parameter, δ, and cost factor, d, provide the necessary information to utilize the Rayleigh model. Lee, Hogue, and Gallagher (1997) demonstrate how a budget profile forecast is determined given an initial total R&D program cost estimate, D, and either the time of peak expenditures, t_{peak}, or the estimated program completion time,

t_{final}. We turn our attention now to the characteristics that limit the Rayleigh model in this application.

The Limitations of the Rayleigh Model. There are two characteristics that theoretically limit the Rayleigh distribution in modeling R&D program expenditures. The Weibull distribution simplifies to the Rayleigh distribution when removing the location parameter, γ, and fixing the shape parameter $\beta = 2$. These two characteristics of the Rayleigh distribution make the Rayleigh model somewhat rigid in its ability to model program expenditures.

The Rayleigh Shape Parameter $\beta = 2$. This characteristic of the Rayleigh distribution results in the peak expenditures occurring approximately at the 38^{th} percentile of the total program duration (Gallagher and Lee, 1996:52). However, a program expenditure rate may peak earlier or later. For example, a program might have expenditures that stop shortly after the time of peak rate of expenditures, indicating a very short tail. In this case the Rayleigh shape parameter, which is fixed at two, causes the Rayleigh distribution to forecast reality inaccurately. The solution to this problem is allowing the shape parameter to vary, which leads us to the Weibull distribution.

The Rayleigh Distribution Cannot Model Insignificant Funding. The Rayleigh model does not have a location parameter, which gives the Weibull distribution flexibility in modeling the relative start of a program. For example, initial R&D programs sometimes start out with a few years of minimal funding. The Rayleigh distribution lacks the ability to model this insignificant funding. The location parameter allows the Weibull distribution to model programs with no funding for the initial years of insignificant funding.

In summary, the Rayleigh function is somewhat rigid in its ability to model R&D expenditures. The varying shape parameter and additional location parameter make the Weibull distribution more flexible in modeling R&D program expenditures. For this reason our discussion now focuses on the Weibull function.

The Weibull Model's Flexibility

When discussing the Weibull verses the Rayleigh model there are two major differences. Notice that (3) and (6) are equivalent with the exception of the location parameter and the fixed shape parameter $\beta = 2$. The location parameter gives flexibility to the Weibull function to model program expenditures when relatively insignificant funding occurs at the beginning of the program. Because the Weibull shape parameter varies, the time of peak expenditures does not fix the program completion time. Since a fixed shape parameter does not accurately model R&D program expenditures as well as one that varies, we use the three-parameter Weibull function allowing the shape parameter to vary. We obtain a better fit of the data using the Weibull model as opposed to the Rayleigh model.

The Weibull function provides the flexibility to estimate various distributions. For example, when the shape parameter, $\beta = 1$, it produces an exponential distribution. When $\beta = 2$, as stated earlier, it produces the Rayleigh distribution. For $\beta = 3.4$, the Weibull is approximately normally distributed. The Weibull shape parameter determines the time of peak expenditures. Figure 6 shows the effects of changing the Weibull shape parameter when holding the location and scale parameters constant.

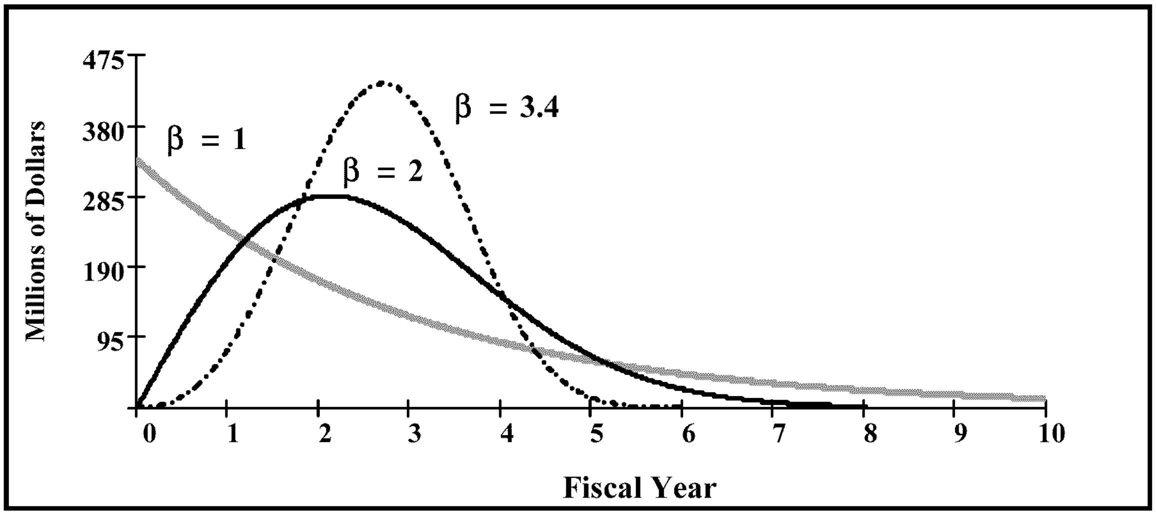

Figure 6. Effects of changing Weibull shape β, with $\gamma = 0$, $\delta = 3$, $d = 1000$

Figure 7 demonstrates the effects of changing the Weibull scale δ parameter while holding the shape and location parameters constant. Increasing the scale parameter extends a program's completion time.

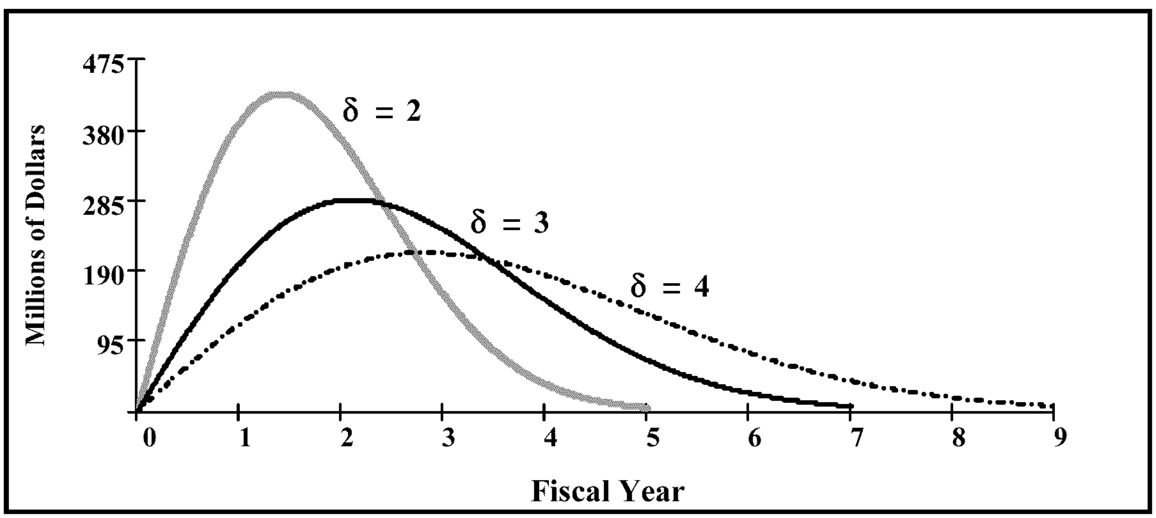

Figure 7. Effects of changing Weibull scale δ, with $\gamma = 0$, $\beta = 2$, $d = 1000$

Figure 8 illustrates the Weibull location parameter's ability to model insignificant funding in the first few years of the program. In essence, the location parameter changes the start time t when R&D program expenditures are significant.

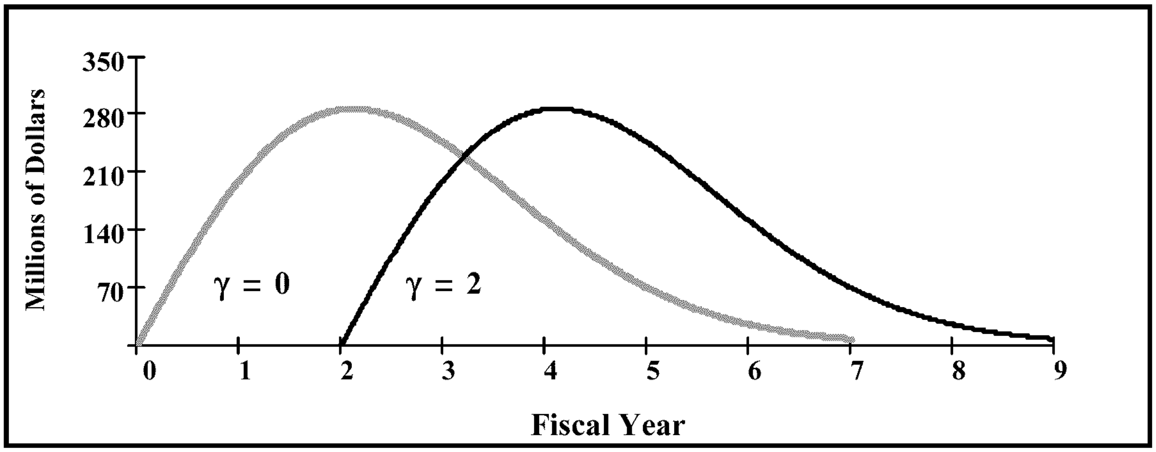

Figure 8. Effects of changing Weibull Location γ, with $\beta = 2$, $\delta = 3$, $d = 1000$

In summary, the characteristics of the Weibull models have an initial ramp-up, a peak, and an exponential tail. The initial ramp up is due to the improvement in performance. The exponential decrease in the tail is caused by the exhaustion of the work, since work performed is proportional to work remaining.

Converting a Point Estimate to a Budget Profile

When Military Departments plan Research and Development (R&D) programs, they must project a budget profile. The budget or total obligation authority (TOA) is the necessary funding to execute the program over multiple years. The OSD comptroller prescribes standard expenditure patterns or percentages of R&D funds to be spent in a given year. The targeted percentage of funds spent in a particular budget year is called an outlay rate. Inflation is also used to calculate the necessary budget requirements in the out years of R&D programs.

The success of a program is determined by several factors. One important factor that this thesis effort hopes to improve is a forecasted budget profile that will better meet

each fiscal year expenditure requirement. The challenge is to ensure sufficient funding in a particular year given each budget year's outlay pattern.

Lee, Hogue, and Gallagher (1997) present a method to derive the appropriate budget for a given total program cost estimate using Rayleigh distributed expenditures. This thesis effort uses their method of forecasting R&D program budgets, but with Weibull distributed expenditures. We present an example of Lee, Hogue, and Gallagher's (1997) method of determining a budget profile using the Rayleigh model when given R&D program estimates for total cost, D, and completion, t_{final}.

We demonstrate a simple hypothetical program with R&D expenditures of 500 million dollars occurring over 10 years. The program is in base year 2000. Given D = 500 and t_{final} = 10, the Rayleigh model parameters calculated with (11) and (12) are d = 515.46 and δ = 5.345. Table 1 column 3 shows the Rayleigh cumulative constant-dollar expenditures, calculated using (8). Since the Rayleigh cumulative expenditures must be converted to annual expenditures, $\hat{O}_i$ for the ith year of the program, we use the formula

$$\hat{O}_i = E(t_i) - E(t_{i-1}), \tag{13}$$

where E is the constant-dollar cumulative expenditures in (8) and time t_i is the fiscal year end i. The hat represents modeled values, different from budget derived values with (1) and (2). Table 1 column 4 shows the annual constant-dollar expenditures and column 5 shows the annual current-year dollars by multiplying the Air Force raw inflation indices, c_i, shown in the last column.

Table 1. Rayleigh-Based Expenditure Profile Example

Fiscal Year	Time	Rayleigh Cumulative Expenditures CY00$M	Rayleigh Annual Expenditures CY00$M	Rayleigh Annual Expenditures Current $M	Air Force Raw Inflation Indices
2000	1	17.73	17.73	17.73	1.0000
2001	2	67.34	49.61	50.36	1.0150
2002	3	139.28	71.94	74.12	1.0302
2003	4	221.03	81.74	85.48	1.0457
2004	5	300.59	79.56	84.86	1.0666
2005	6	369.25	68.66	74.70	1.0879
2006	7	422.70	53.45	59.31	1.1097
2007	8	460.59	37.89	42.89	1.1319
2008	9	485.20	24.61	28.41	1.1545
2009	10	499.90	14.70	17.31	1.1776
Total			499.90	535.16	

To this point of the example, several key points are made. The Rayleigh cumulative expenditures show the necessary funding for the program's success in constant dollars. The Rayleigh annual expenditures in constant dollars provide the funding needed in base year 2000. The Rayleigh annual expenditures in current dollars provide the funding needed in the future years, taking into account inflation. For example in Table 1, in FY2009, the program needs $14.7M, expressed in FY2000 dollars or equivalently $17.31M expressed in FY2009 dollars. For the program manager to meet program expenditure requirements, the sum of previous and current budgets multiplied by the outlay rates should equal the annual expenditures in current-year dollars.

The developed expenditure profile provides how much the program will spend each year. The task is to determine a budget profile that will meet a program's expenditure requirements. There is a difference between a budget profile and an expenditure profile. The budget profile contains the necessary funds for the life of the

program taking into account the OSD Comptroller outlay rates and inflation. The expenditure profile is comprised of a mixture of different years of budgeted funds according to the outlay percentages.

The example continues with our approach to develop the necessary TOA or budget profile to meet fiscal expenditure requirements. To develop a budget profile we must take into account that the total appropriation for an R&D program cannot be spent during the year in which it is authorized. The outlay rates govern the amount of yearly budget funds a program is allotted each year.

For our example we use hypothetical OSD comptroller published outlay rates of 0.540, 0.324, 0.090, 0.020, 0.011, 0.004, and 0.002 for the first through seventh year respectively. This would suggest that if a program had \$10M appropriated to it in the first year, only \$5.4M should be expended. The second year the program manager should expend \$3.24M of the first year's budget authority. If the expenditure requirements for the second year of the program are \$15M, then the program manager should ask for \$21.78M in appropriations for the second year. The example calculation is provided (\$15M needed - \$3.24M Yr-1 appropriation)/(.5400 Yr-1 outlay rate) = \$21.78M.

The complexity of developing a budget profile compounds according to the duration and size of the program. The expenditure profile is easily forecasted with the Rayleigh and Weibull models. The challenge is converting that expenditure profile to a budget profile when taking into account relevant appropriation years related to different outlay rates. Lee, Hogue, and Gallagher (1997) provide a nonlinear estimation approach, which is our final step in forecasting an R&D program budget profile.

This method allows the budget estimate, $\hat{B}_i$ for each budget year *i*, to change simultaneously until an optimal solution is reached. We apply this approach by substituting the $\hat{B}_i$ in (1) and (2) and allow Microsoft Excel Solver function (2000) to select the yearly budget estimates that minimize the sum of squared errors between the Rayleigh modeled expenditures and the actual expenditures that result from the forecasted budget profile. Specifically,

$$\min \sum_{i=1}^{N+J-1} (\tilde{O}_i - \hat{O}_i)^2 \tag{14}$$

where $\hat{B}_i \geq 0$ and $\hat{O}_i$ is the Rayleigh modeled expenditure profile (13) and $\tilde{O}_i$ is the actual expenditure profile for each *i*th year from (1) and (2). *N+J-1* represents the total program and outlay years to calculate. Table 2 shows the results of our example program with a generated budget profile in the final column.

Table 2. Rayleigh-Based Budget Profile Projection Example

Fiscal Year	**Time**	**Annual Current $M Expenditures**		**Error**	**(Error)^2**	**Budget Profile Current $M**
		Rayleigh	**Estimated**			
2000	1	17.73	17.75	-0.02	0.00	34.43
2001	2	50.36	50.32	0.04	0.00	73.13
2002	3	74.12	74.18	-0.06	0.00	87.14
2003	4	85.48	85.37	0.11	0.01	91.72
2004	5	84.86	85.04	-0.18	0.03	83.32
2005	6	74.70	74.41	0.29	0.09	66.49
2006	7	59.31	59.81	-0.50	0.25	50.61
2007	8	42.89	42.01	0.88	0.78	29.97
2008	9	28.41	29.92	-1.51	2.27	24.38
2009	10	17.31	14.33	2.99	8.92	2.12
2010	11	0.00	3.94	-3.94	15.53	0.00
2011	12	0.00	1.21	-1.21	1.46	0.00
2012	13	0.00	0.35	-0.35	0.12	0.00
2013	14	0.00	0.09	-0.09	0.01	0.00
2014	15	0.00	0.01	-0.01	0.00	0.00
Total		535.16	538.72	-3.56	29.47	543.30

Chapter Summary

Based on past research, the Rayleigh function provides a model to determine R&D expenditure profiles. This chapter details these fundamental assumptions starting with Norden who used the Rayleigh to model manpower and development effort over time. Putnam derived cumulative software costs using the Rayleigh model, and Lee, Hogue, and Gallagher employed the Rayleigh to model defense development expenditures. We justify using the Weibull function in modeling expenditure profiles according to the generalizations of the Rayleigh model. Following the description of the parameter characteristics, and limitations of the Rayleigh function, we focus on the flexible Weibull function. After deriving the Weibull function we explain the parameters and discuss its ability to model R&D program expenditures. A brief discussion of Lee, Hogue, Gallagher's method of forecasting a budget profile from an R&D cost estimate concludes this chapter.

III. Research Methodology

Chapter Overview

Past research shows that the Rayleigh cumulative distribution function models R&D program expenditures well. The Weibull function models R&D program expenditures better than the Rayleigh. This research employs the Weibull function to forecast an initial R&D budget profile by developing regression models to predict the necessary Weibull shape and scale parameters. A description of the methodology to develop the predictive shape and scale regression models to forecast Weibull-based budgets is the focus of this chapter.

Appropriate data collection is the initial step in this research. The Selected Acquisition Report (SAR), maintained by the Office of the Secretary of Defense is our source of data. For each R&D program we collect various categorical characteristics and the final annual budget profile data. After we convert each budget profile in current dollars to expenditure profiles in fiscal year 2000 constant dollars, we estimate the Weibull shape and scale parameters. We test the assumption that the Weibull distribution fits program expenditures using Komolgorov-Smirnov, Cramer-von Mises, and Anderson-Darling goodness-of-fit (GOF) statistics. The least squares estimated shape and scale parameters are the responses or dependent variables we predict in our regression models. The final cost and schedule with various categorical data like branch of military service and type of program provide possible predictors or independent variables in our regression models.

We randomly select 80 percent of the full data set to build our Weibull shape and scale regression models, and set aside the remaining 20 percent for validation. We test for a mathematical relationship between the Weibull shape and scale parameters against possible predictors like military branch of service, program type, total cost and duration. Using multiple regression we develop predictive models for the Weibull shape and scale parameters. To validate the robustness of our resulting regression models, we determine if the remaining 20 percent completed R&D program least squares estimated shape and scale validation data values fall within a 95 percent prediction interval. To determine the Weibull model's budget forecasting capability, we compare forecasted Weibull-based budgets to 128 completed R&D program budgets using Lee, Hogue, and Gallagher's (1997) methodology. Using the same methodology we determine the significance of our results by comparing the average correlation of forecasted Weibull-based and Rayleigh-based budgets to the same 128 completed program budgets.

<u>Program Data Collection</u>

We gather R&D program funding data from the Selected Acquisition Reports (SARs) to evaluate our research hypothesis. Our data collection comprises final Research, Development, Test and Evaluation program annual budget profiles to Milestone III or equivalent, military branch of service, program type, and the final program cost and duration. Since the SAR did not report annual budget data prior to 1982, our data collection includes only those programs with a final SAR report dated 1982 or later. The SAR presents the budget profiles in millions of both constant and current-year dollars. We collect only current-year budget profiles for the purpose of

converting current to constant-year dollars consistently across programs. Our selection criteria include programs that were not terminated with at least a three-year budget profile to Milestone III. Our full data set comprises 128 R&D programs. Table 3 presents the Air Force's Airborne Warning and Control System (AWACS) Radar System Improvement Program (RSIP) as an example final SAR budget profile. We graphically display the RSIP final program budget profile in Figure 9.

Table 3. RSIP Final Program Budget Profile

Fiscal Year	1989	1990	1991	1992	1993	1994	1995	1996	Total
Current $M	44.2	63.7	71.8	117.1	15.4	38.4	42.7	31.1	424.4

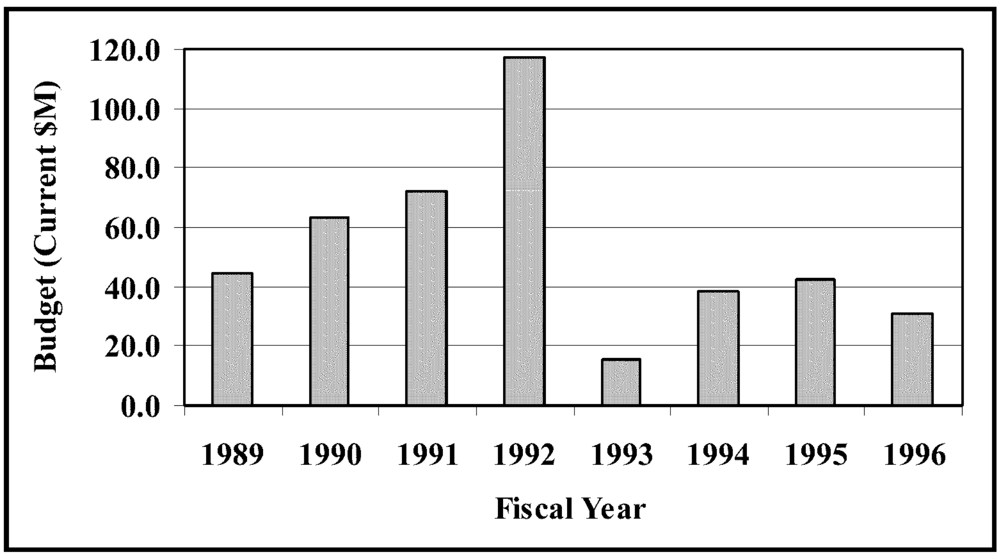

Figure 9. RSIP Final Budget Profile

<u>Convert Budget Profiles to Expenditure Profiles</u>

We convert the program budget data into constant-dollar expenditures in two steps: 1) convert the budget profile into an expenditure profile and 2) adjust for inflation. The first step is to convert the current-dollar budget profile into current-dollar

expenditures. We achieve this conversion using (1) when multiplying each budget year by the appropriate outlay rates and summing the expended funds for each fiscal year. Since the yearly outlay rates vary slightly from year to year, we use average outlay rates from 1993 to 2001 for our calculations.

Table 4. Average Service R&D Outlay Rates (as Percentages)

Air Force											
	FY01	FY00	FY99	FY98	FY97	FY96	FY95	FY94	FY93	**Avg**	StDev
Yr1	59.5	58.8	59.1	50.7	45.8	46.3	46.5	46.5	50.8	**51.56**	5.98
Yr2	33.7	34.5	33.1	37.4	39.9	39.1	38.8	38.8	34.5	**36.64**	2.67
Yr3	3.6	3.6	5.3	6.8	8.9	8.9	8.8	8.9	9.5	**7.14**	2.40
Yr4	1.0	1.0	1.4	3.0	3.6	3.6	3.6	3.6	3.4	**2.69**	1.19
Yr5	0.3	0.3	0.4	0.8	1.1	1.1	1.1	1.1	1.1	**0.81**	0.37
Yr6	0.0	0.3	0.3	0.3	0.3	0.4	0.4	0.4	0.4	**0.31**	0.13
Yr7	0.0	0.0	0.2	0.0	0.1	0.0	0.2	0.0	0.2	**0.08**	0.10
	98.1	98.5	99.8	99.0	99.7	99.4	99.4	99.3	99.9	**99.23**	
Army											
	FY01	FY00	FY99	FY98	FY97	FY96	FY95	FY94	FY93	**Avg**	StDev
Yr1	57.5	56.8	58.0	58.0	58.0	57.0	57.0	55.0	55.0	**56.92**	1.19
Yr2	32.5	33.7	33.0	33.0	33.0	34.0	34.0	34.0	34.0	**33.47**	0.59
Yr3	6.3	5.0	5.3	5.3	5.3	5.3	5.3	7.3	7.3	**5.82**	0.91
Yr4	2.1	2.1	2.1	1.8	1.8	1.8	1.8	1.8	1.8	**1.90**	0.15
Yr5	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	**0.80**	0.00
Yr6	0.0	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	**0.44**	0.17
Yr7	0.0	0.0	0.2	0.0	0.2	0.0	0.2	0.0	0.2	**0.09**	0.11
	99.2	98.9	99.9	99.4	99.6	99.4	99.6	99.4	99.6	**99.44**	
Navy											
	FY01	FY00	FY99	FY98	FY97	FY96	FY95	FY94	FY93	**Avg**	StDev
Yr1	59.5	59.3	60.5	58.0	55.9	55.9	54.0	55.0	55.0	**57.01**	2.35
Yr2	31.4	33.6	32.5	33.1	31.5	31.5	32.4	33.4	33.4	**32.53**	0.89
Yr3	5.9	4.5	4.5	5.4	8.2	8.2	8.0	7.8	7.8	**6.70**	1.61
Yr4	1.9	1.0	1.0	2.0	2.0	2.0	2.0	1.3	1.3	**1.61**	0.45
Yr5	0.7	0.3	0.3	0.6	1.1	1.1	1.1	1.1	1.1	**0.82**	0.35
Yr6	0.0	0.2	0.2	0.2	0.4	0.4	0.4	0.4	0.4	**0.29**	0.15
Yr7	0.0	0.0	0.0	0.0	0.6	0.0	0.2	0.0	0.2	**0.11**	0.20
	99.4	98.9	99.0	99.3	99.7	99.1	98.1	99.0	99.2	**99.08**	

Table 4 shows the average outlay rates and the standard deviation for each military branch of service. Table 5 gives an illustration of our calculations.

Table 5. RSIP Final Budget Converted to Expenditures (Current $M)

	Average Air Force Outlay Rates (1993-2001)					
Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Rate	0.5156	0.3664	0.0714	0.0269	0.0081	0.0031

FY	Budget	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001
1989	44.2	22.8	16.2	3.2	1.2	0.4	0.1							
1990	63.7		32.8	23.3	4.6	1.7	0.5	0.2						
1991	71.8			37.0	26.3	5.1	1.9	0.6	0.2					
1992	117.1				60.4	42.9	8.4	3.1	0.9	0.4				
1993	15.4					7.9	5.6	1.1	0.4	0.1	0.0			
1994	38.4						19.8	14.1	2.7	1.0	0.3	0.1		
1995	42.7							22.0	15.6	3.1	1.1	0.3	0.1	
1996	31.1								16.0	11.4	2.2	0.8	0.3	0.1
Expenditures		22.8	49.0	63.5	92.4	58.1	36.4	41.1	36.0	16.0	3.7	1.3	0.4	0.1

The final step converts the program current-dollar expenditures shown as the bottom line in Table 5, to constant–dollar expenditures by removing the effects of inflation. Table 6 shows the current to constant dollar conversion with (2). The current-dollar expenditures in row 2 are divided by the raw inflation indices in row 3, producing the desired constant-dollar expenditures in row 4. Figure 10 graphically illustrates the final budget profile, expenditure profile in current dollars, and the expenditure profile in constant dollars for the RSIP program.

Table 6. RSIP Current $M Expenditures to Constant $M Expenditures

Fiscal Year	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001
Exp. (Current $M)	22.8	49.0	63.5	92.4	58.1	36.4	41.1	36.0	16.0	3.7	1.3	0.4	0.1
Inflation Factor	0.787	0.818	0.854	0.877	0.901	0.919	0.937	0.955	0.975	0.982	0.990	1.000	1.015
Exp. (Constant $M)	29.0	59.9	74.4	105.3	64.4	39.6	43.9	37.7	16.4	3.8	1.3	0.4	0.1

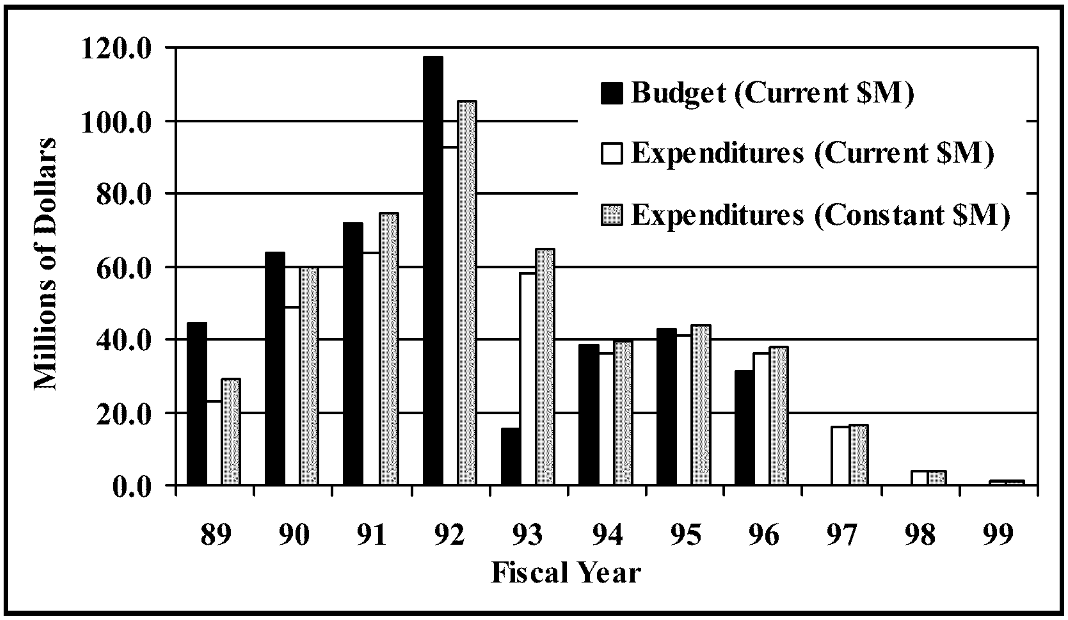

Figure 10. RSIP Budget and Expenditure Profiles

Table 7 provides a summary of our conversion calculations with (1) and (2).

Table 7. RSIP Budget to Expenditures Summary Conversion

Fiscal Year	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001
Budget (Current $M)	44.2	63.7	71.8	117.1	15.4	38.4	42.7	31.1					
Exp. (Current $M)	22.8	49.0	63.5	92.4	58.1	36.4	41.1	36.0	16.0	3.7	1.3	0.4	0.1
Exp. (Constant $M)	29.0	59.9	74.4	105.3	64.4	39.6	43.9	37.7	16.4	3.8	1.3	0.4	0.1

In summary, we convert a budget profile to an expenditure profile in constant dollars. We now turn our attention to the estimation of the Weibull parameters. We use the least squares error method by fitting the Weibull cumulative distribution of expenditures to the RSIP cumulative constant dollar expenditures. We test our assumption that R&D program expenditures are Weibull distributed using GOF statistics.

Estimate Weibull Parameters

Nonlinear estimation is the method we used to estimate the Weibull shape, scale, and location parameters in (5). We allow the Weibull parameters to vary until we minimize the sum-squared difference between the actual cumulative constant-dollar expenditures and the Weibull modeled cumulative constant-dollar expenditures. From a budget profile, the actual cumulative constant-dollar expenditures are calculated with (1) and (2). The Weibull cumulative constant-dollar expenditures are calculated with (5).

We use Microsoft Excel's Solver package (2000) as our nonlinear estimation tool. The target cell in Solver is set to minimize the sum of all the errors squared. The Weibull shape, scale, and location parameters are set as the changing cells. Solver determines the optimal values for the Weibull shape, scale, and location parameters that minimize the total sum-squared errors with (14).

Table 8 shows the conversion process from the reported final budget to cumulative constant-dollar expenditures for the Air Force RSIP program. The final budget profile, conversion to current-dollar expenditures, conversion to constant-dollar expenditures, and resulting cumulative constant-dollar expenditures are displayed in rows 2, 3, 4, and 5 respectively.

Table 8. RSIP Cumulative Constant-Dollar Expenditure Profile

Fiscal Year	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001
Budget (Current $M)	44.2	63.7	71.8	117.1	15.4	38.4	42.7	31.1					
Exp. (Current $M)	22.8	49.0	63.5	92.4	58.1	36.4	41.1	36.0	16.0	3.7	1.3	0.4	0.1
Exp. (Constant $M)	29.0	59.9	74.4	105.3	64.4	39.6	43.9	37.7	16.4	3.8	1.3	0.4	0.1
Cum. Exp. (Constant $M)	29.0	88.9	163.3	268.6	333.0	372.6	416.5	454.2	470.6	474.4	475.7	476.1	476.2

We now estimate the Weibull parameters that minimize the sum-squared errors. Table 9 presents the minimization outcome for the RSIP program. The total sum-squared error is greater than 510.1 for any other combination of Weibull parameters.

Table 9. RSIP Minimized $\Sigma(\text{errors})^2$

Fiscal	Cumulative CY00$M Expenditures			
Year	Actual	Weibull	Error	$(\text{Error})^2$
1989	29.0	24.6	4.4	19.2
1990	88.9	91.4	-2.5	6.5
1991	163.3	174.0	-10.7	113.8
1992	268.6	255.6	13.0	169.1
1993	333.0	326.5	6.6	43.1
1994	372.6	382.3	-9.6	92.9
1995	416.5	422.8	-6.3	39.8
1996	454.2	450.4	3.9	15.0
1997	470.6	467.9	2.7	7.3
1998	474.4	476.2	-1.8	3.2
1999	475.7	476.2	-0.5	0.2
2000	476.1	476.2	-0.1	0.0
2001	476.2	476.2	0.0	0.0
			$\Sigma(\text{errors})^2$ =	**510.1**

Table 10 presents the estimated Weibull parameters that produce our minimization results. The cost factor is equal to the final cost of the program divided by 0.97. The initial search values for the shape and scale parameters are set greater than 0.1. The location parameter is set to zero to represent presumed immediate program start.

Table 10. RSIP Least Squares Estimated Weibull Parameters

Cost Factor	Scale δ	Shape β	Location γ
490.93	4.540	1.694	0.213

Figure 11 demonstrates the resulting Weibull modeled constant-dollar expenditures fit to the cumulative constant-dollar expenditures.

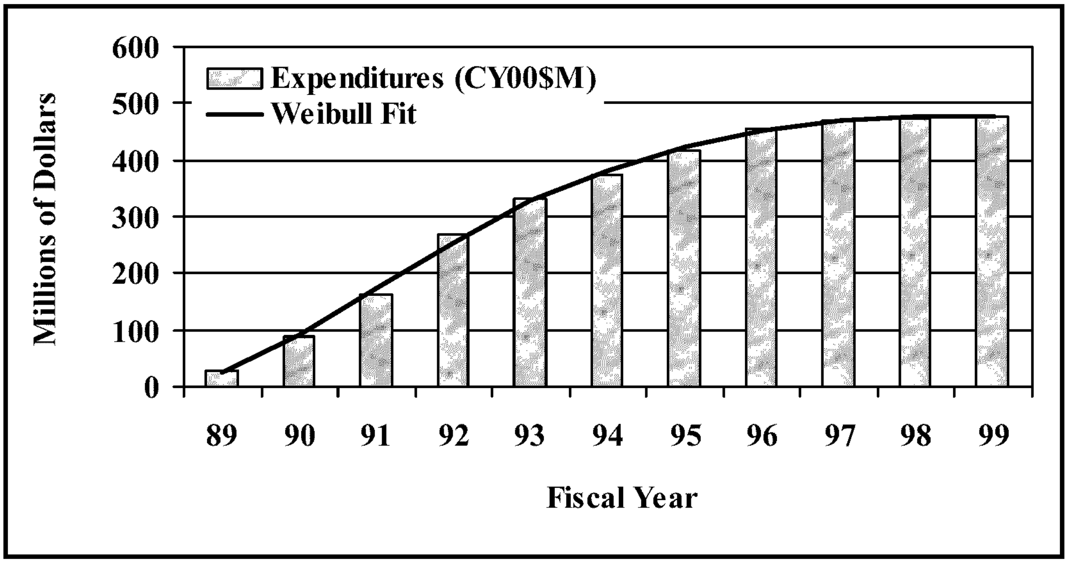

Figure 11. RSIP Cumulative Least Squares Weibull Fit

Figure 12 shows the resulting Weibull modeled constant-dollar expenditure fit to the cumulative constant-dollar expenditures converted to annual expenditures.

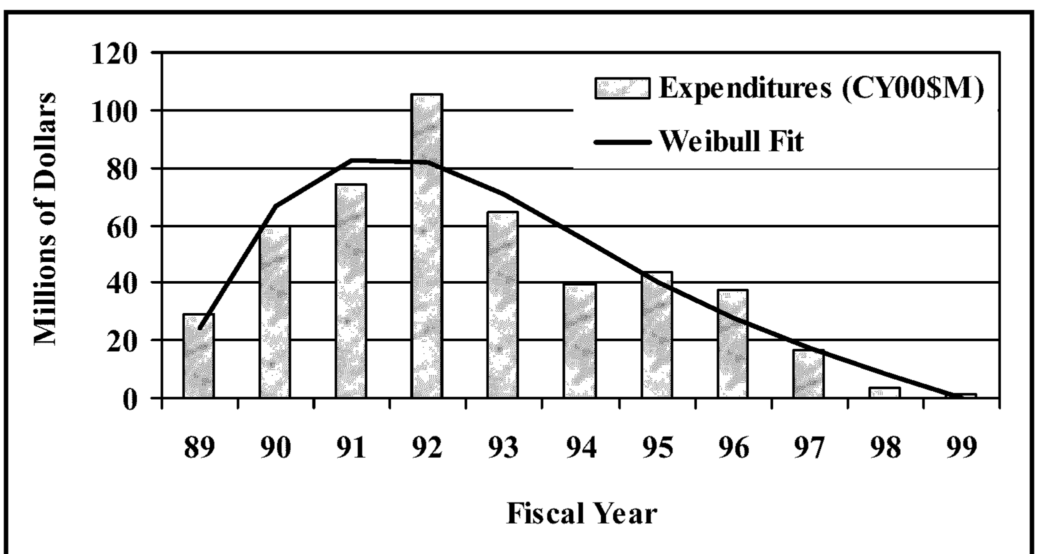

Figure 12. RSIP Cumulative Least Squares Weibull Fit Converted to Annual

In summary, minimizing the total sum-squared difference between the actual cumulative constant-dollar expenditures and the Weibull modeled cumulative constant-dollar expenditures provides the Weibull estimated parameters. The estimated Weibull shape and scale parameters provide the responses or dependent variables for our regression analysis. The categorical and the final cost and schedule data provide the predictors or independent variables for our regression model.

Perform Goodness-of-Fit (GOF) Tests

Porter (2001) and Unger (2001) find that the Weibull model better fits R&D program expenditures. The measurements we use to evaluate if expenditures follow the theoretical Weibull are three goodness-of-fit (GOF) statistics, the Komolgorov-Smirnov (K-S), Cramer-von Mises (CvM), and Anderson-Darling (A-D). Unger (2001) modifies each of these statistics to perform GOF tests on discrete distributions. Unger statistically compares the deviation between $F_n(x)$, which is the empirical distribution function (EDF) based on n data points, and $F_o(x)$, which is the hypothesized CDF. Using Unger's application, the EDF is the cumulative constant-year expenditures reported at the end of each fiscal year, from (13), divided by the cost factor, $F_n(i) = C_i / d$. The hypothesized model is (1) with the least squares estimates for the Weibull from (14). Unger defines the K-S, CvM, and A-D GOF statistics as

$$K = \max\left|F_n(i) - F_o(i)\right| \text{ for } i = 1,2,\ldots,N+J-1, \tag{15}$$

$$AD = n\sum_{i=1}^{i=n}[\,(F_n(i))^2(\ln(F_o(i+1)) - \ln(F_o(i))) -$$

$$(F_n(i-1)-1)^2(\ln(1-F_o(i)) - \ln(1-F_o(i-1)))] - n, \tag{16}$$

and

$$W^2 = \frac{n}{3}\sum_{i=1}^{i=n+1}\left[\left(F_0(i) - F_n(i-1)\right)^3 - \left(F_0(i-1) - F_n(i-1)\right)^3\right] \qquad (17)$$

respectively. Unger provides a description and the derivations for the K-S, CvM, and A-D GOF statistics.

Regression Analysis

We use multiple regression to determine if there is a mathematical relationship between the necessary Weibull parameters and possible predictors that a new start R&D program possesses. To build the Weibull shape and scale regression models, we select two categorical (nominal) and two numerical (continuous) predictor variables. The two categorical predictors are program lead branch of service (Air Force, Army, and Navy) and program system type (Aircraft, Electronic, Missile, Munitions, Ship, Space, and Vehicle). The lead service for joint programs is determined by the service that contributed the highest dollar value to the R&D effort. The two numerical predictors are the program total 2000 constant dollars (CY00$) and the program duration in years to Milestone III. Using the predictive values from our shape and scale regression models, we apply the Weibull model to forecast R&D program budget profiles. This leads to the final section of our methodology of relating the Weibull model to budget profiles.

Convert Expenditure Profiles to Budget Profiles

The shape and scale are the necessary parameters to utilize the Weibull model to forecast R&D program budget profiles. The Weibull shape and scale predictive models we obtain from our regression analysis provide the necessary predicted parameters to

employ the Weibull model. Lee, Hogue, and Gallagher (1997) forecast a budget profile from a point estimate using Rayleigh-based expenditures. We employ their method using Weibull-based expenditures.

Chapter Summary

This chapter explains the proposed methodology to predict the requisite shape and scale parameters to forecast Weibull-based initial R&D program budget profiles. We search for relationships to predict the Weibull shape and scale parameters using multiple regression analysis. We convert 128 program's current-dollar budget profiles to fiscal year 2000 constant dollar cumulative expenditure profiles. To build the response portion of our model data, we estimate the shape and scale parameters by minimizing the sum errors squared between the actual program cumulative expenditures and the Weibull modeled cumulative expenditures. We randomly selected 102 (80%) of the programs to perform our regression analysis, and the remaining 26 (20%) are saved for model validation. Finally, we forecast Weibull-based R&D program budget profiles by using the predicted shape and scale values from our regression models. In Chapter Four, we provide the results of our methodology application to completed R&D programs.

IV. Research Results

Chapter Overview

Chapter Three outlined the methodology to predict shape and scale parameters requisite to forecast Weibull-based R&D budget profiles. This chapter presents our research results when applying the 128 completed R&D defense programs to our methodology. We present the goodness-of-fit (GOF) results in support of our assumption that the Weibull model fits R&D program expenditures. We present our Weibull shape and scale predictive models, and demonstrate that both regression models are statistically significant. Using our predictive shape and scale models to utilize the Weibull function, we demonstrate the Weibull model's ability to forecast budget profiles. Finally, we demonstrate the improved forecasting ability of the Weibull model when compared to that of the Rayleigh model.

Goodness of Fit Results

The three GOF statistics we use to determine if the 128 R&D program expenditures are Weibull distributed are the Komolgorov-Smirnov (K-S), Cramer-von Mises (CvM), and Anderson-Darling (A-D). The test results are separated by the program budget duration in years. Programs that are accepted represent expenditures that are Weibull distributed. Tables 11-13 show the results for the K-S (89% accept), CvM (71% accept), and A-D (72% accept) GOF tests performed for all 128 programs respectively.

Table 11. Komolgorov-Smirnov Goodness-of-Fit Results

Program Duration	Programs	Accept	Reject	% Accept	% Reject
Duration $\leq$ 3	5	3	2	60%	40%
3 < Duration $\leq$ 4	17	11	6	65%	35%
4 < Duration $\leq$ 5	15	14	1	93%	7%
5 < Duration $\leq$ 6	14	14	0	100%	0%
6 < Duration $\leq$ 7	14	12	2	86%	14%
7 < Duration $\leq$ 22	63	60	3	95%	5%
Total	128	114	14	**89%**	11%

Table 12. Cramer-von Mises Goodness-of-Fit Results

Program Duration	Programs	Accept	Reject	% Accept	% Reject
Duration $\leq$ 3	5	0	5	0%	100%
3 < Duration $\leq$ 4	17	1	16	6%	94%
4 < Duration $\leq$ 5	15	7	8	47%	53%
5 < Duration $\leq$ 6	14	10	4	71%	29%
6 < Duration $\leq$ 7	14	13	1	93%	7%
7 < Duration $\leq$ 22	63	60	3	95%	5%
Total	128	91	37	**71%**	29%

Table 13. Anderson-Darling Goodness-of-Fit Results

Program Duration	Programs	Accept	Reject	% Accept	% Reject
Duration $\leq$ 3	5	0	5	0%	100%
3 < Duration $\leq$ 4	17	7	10	41%	59%
4 < Duration $\leq$ 5	15	11	4	73%	27%
5 < Duration $\leq$ 6	14	8	6	57%	43%
6 < Duration $\leq$ 7	14	10	4	71%	29%
7 < Duration $\leq$ 22	63	56	7	89%	11%
Total	128	92	36	**72%**	28%

Table 14 shows 77 percent of the 128 program expenditure profiles are Weibull distributed. The GOF test results to determine if R&D program expenditures are Weibull

distributed appears lower than expected; however, annual R&D program expenditures represent a discrete distribution. Many continuous GOF tests fail discrete distributions because of the infinite number of points between each discrete value. Unless the discrete distribution is highly populated as to approach a continuous distribution the GOF test will fail. Unger (2001) adjusts the K-S, CvM, and A-D formulas to perform GOF statistics on program expenditures that are discretely distributed. The GOF tests do not penalize the expenditure profile any more or less than the previous expenditure year. This in essence converts the expenditure profile into a rigid continuous distribution increasing or decreasing with each expenditure year. The more rigid the expenditure profile the less likely the program expenditures are Weibull distributed. Smoother expenditure profiles are more likely for programs with many expenditure years, thus we expect higher failure rates for smaller program durations.

Table 14. Overall Goodness-of-Fit Test Results

Test Type	Accept	Reject	% Accept	% Reject
Komolgorov-Smirnov (K-S)	114	14	89%	11%
Cramer-von Mises (CvM)	91	37	71%	29%
Anderson-Darling (A-D)	92	36	72%	28%
Total	297	87	**77%**	23%

The GOF results show high failure rates for the 51 programs with duration of six years or less. Table 15 indicates that only 56 percent of these 51 program expenditures are Weibull distributed. However, if monthly expenditure data were available for the 51 programs with duration six years or less, we contend that 90 percent of the program monthly expenditures are Weibull distributed. Table 16 indicates that 91 percent of the 77 programs with duration greater than six years are Weibull distributed.

Table 15. Goodness-of-Fit Results (Budget Profiles <= 6 Years)

Test Type (51 Programs)	Accept	Reject	% Accept	% Reject
Komolgorov-Smirnov (K-S)	42	9	82%	18%
Cramer-von Mises (CvM)	18	33	35%	65%
Anderson-Darling (A-D)	26	25	51%	49%
Total	86	67	**56%**	44%

Table 16. Goodness-of-Fit Results (Budget Profiles > 6 Years)

Test Type (77 Programs)	Accept	Reject	% Accept	% Reject
Komolgorov-Smirnov (K-S)	72	5	94%	6%
Cramer-von Mises (CvM)	73	4	95%	5%
Anderson-Darling (A-D)	66	11	86%	14%
Total	211	20	**91%**	9%

The GOF statistics supports the assumption that R&D program expenditures are Weibull distributed. The next section reports the results of our regression analysis to test for relationships that predict the Weibull scale and shape parameters.

Weibull Scale and Shape Regression Models

Research and Development (R&D) defense program expenditures are Weibull distributed. To apply Lee, Hogue, and Gallagher's (1997) method to forecast a budget profile from a point estimate using the Weibull model, we must predict the scale and shape parameters with information known to new start R&D programs. Appendix A shows the 128 completed R&D programs we use to determine a relationship that predicts the necessary Weibull scale and shape parameters. This section of the results will present the scale model, the shape model, and the validation results in predicting the scale and shape least squares parameter estimates.

Regression Analysis Setup. We randomly select 102 (80%) of the total 128 programs to test for mathematical relationships for each of the Weibull scale and shape parameters. The response or dependent variables for each regression model are the Weibull shape and scale least squares parameter estimates from Solver. We use four possible predictors or independent variables. Branch of service and program system type are the first two categorical predictors converted from nominal data to continuous data via indicator variables. We set Air Force as the baseline and convert the branch of service column data into two separate predictor columns as indicator variables for Army and Navy. For the Army data column, we assign a one for Army programs and a zero for Air Force and Navy programs. For the Navy data column, we assign a one for Navy programs and a zero for Air Force and Army programs. We set Vehicle as the baseline for the second categorical predictor, program system type. We establish a data column for each of the remaining program system type categories (Aircraft, Electronic, Missile, Munitions, Ship and Space) in the same fashion as Army and Navy data columns. Program total cost in fiscal year 2000 constant dollars divided by 0.97 (cost factor, *d*) and program duration in total budget years minus the location estimate are the final two continuous predictor variables. We use multiple regression to find a relationship for the Weibull scale and shape parameters.

The Weibull Scale Model. For the scale model results, we display graphically the least squares estimated scale via Solver by the model predicted scale. Statistically, we present the summary of fit, analysis of variance (ANOVA), and parameter model estimates. Finally, we discuss the scale model as a statistically significant predictor for the Weibull scale parameter.

Figure 13 displays the least squares estimated scale by the model's predicted scale. The figure clearly shows that the least squares estimated scale values are tightly arranged along the model's predicted scale regression line, indicating that our regression model predicts scale well.

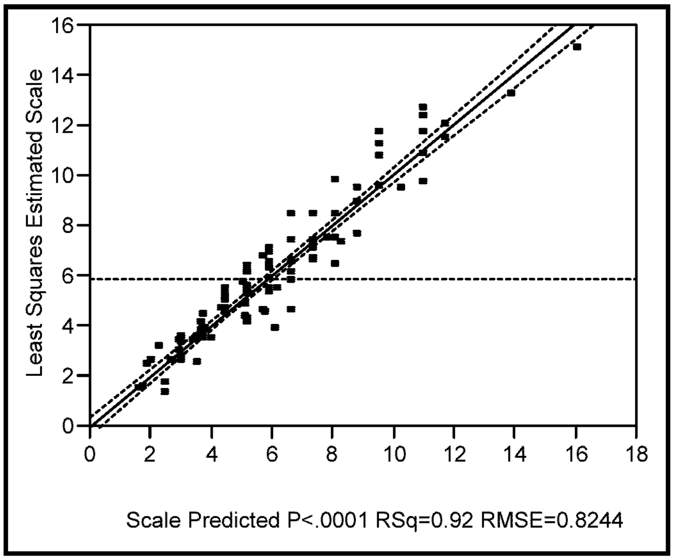

Figure 13. Least Squares Estimated Scale by Predicted Scale Plot

We select the adjusted R^2 over R^2 as the statistical measure of the model's ability to predict the response (least squares estimated scale). The adjusted R^2 compares across models with different numbers of parameters by using the degrees of freedom in its computation (Sall, Lehman, and Creighton, 2001). In our analysis of several models we find that no additional predictor variables or interactions, other than program duration, significantly improve the adjusted R^2 of 0.921.

The ANOVA displays the overall F test, indicating a significant model. When the significance level or *p*-value is less than 0.05 the overall model is significant, indicating that at least one predictor is significant. Because program duration is our only predictor of scale, the t-test, which measures the significance of each predictor in the model, will

mirror the overall F test. Table 17 shows the summary of fit, the ANOVA, and parameter estimates. We define our final scale regression model, δ-hat, as

$$\hat{\delta} = 0.726(Duration)\,. \qquad (18)$$

Table 17. Scale Model Table of Statistics

Scale Model Summary of Fit	
RSquare	0.921671
RSquare Adj	**0.920888**
Root Mean Square Error	0.824422
Mean of Response	5.854373
Observations (or Sum Wgts)	102

Scale Model Analysis of Variance (ANOVA)				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	799.75149	799.751	1176.672
Error	100	67.96724	0.680	**Prob > F**
C. Total	101	867.71873		**<.0001**

Scale Model Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.0683049	0.187391	0.36	0.7163
Duration	0.7256199	0.021153	34.30	**<.0001**

To test the statistical soundness of our scale regression model, we look for possible influential data points that bias what explanatory variables are selected, and test the model assumptions of normality, constant variance, and independence.

Scale Model Influential Data Test. We test for possible influential data points by plotting Cook's D influence statistic. Cook's D influential statistic determines if an observation has a large effect on parameter estimates. Values greater than 0.5 are considered significant influential observations (Neter, Kutner, Nachtsheim, and

Wasserman, 1996:380). Figure 14 displays the overlay plot of values for Cook's D influential statistic, indicating that no observations are significantly influential.

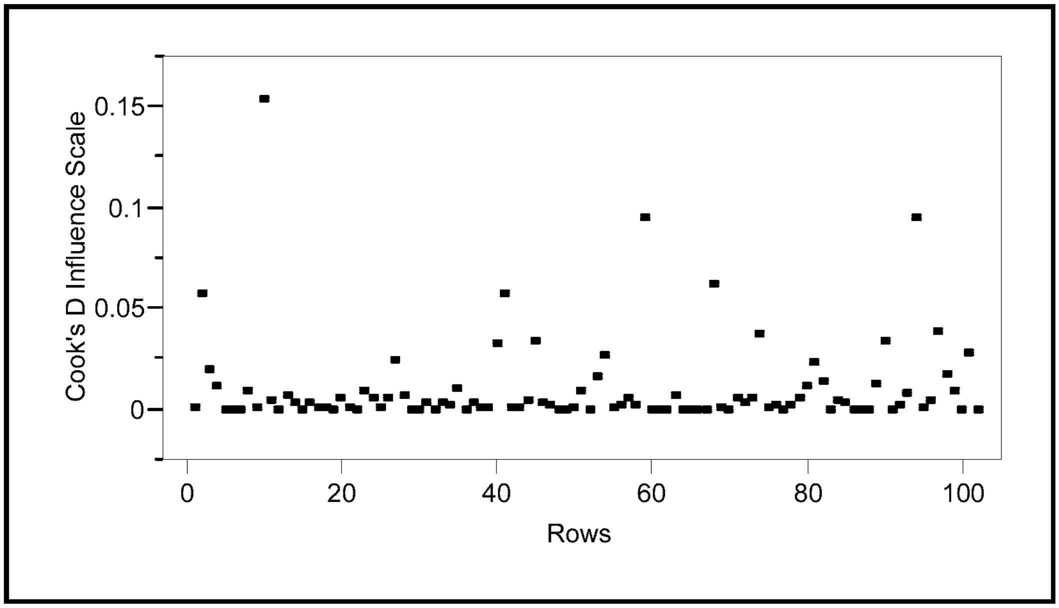

Figure 14. Scale Model Test for Influential Data Points

Scale Model Normality Test. We test for normality by performing a GOF test to determine if the residuals from our regression model are normally distributed. Figure 15 shows the normal distribution fit to the residual distribution and displays the normality test results. If the *p*-value is greater than 0.05 then the residuals are normally distributed. We display a *p*-value of 0.901 indicating that the model residuals are normally distributed.

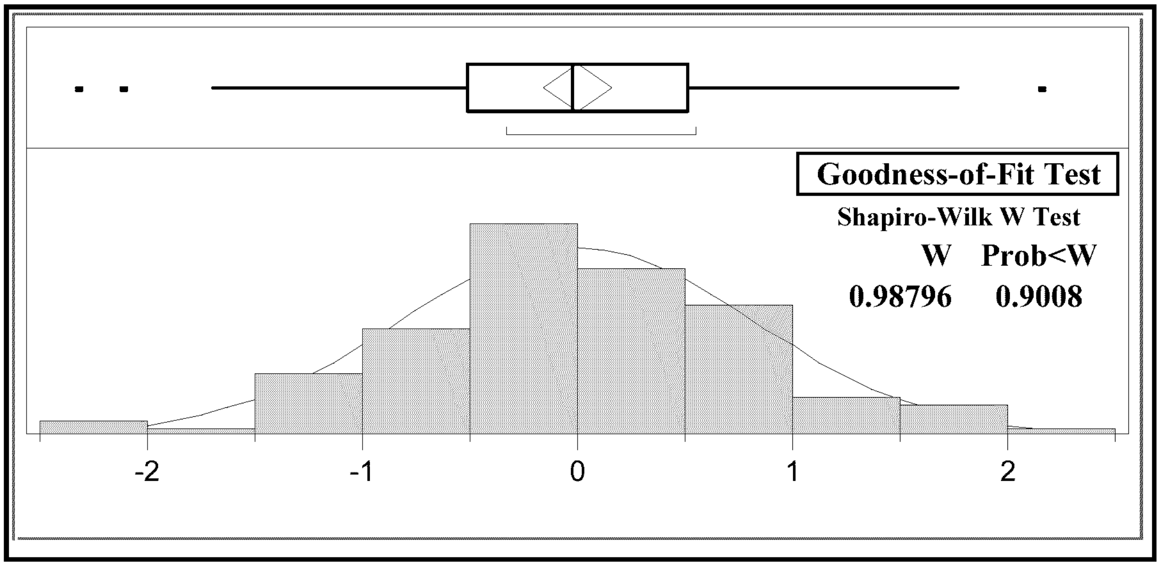

Figure 15. Scale Model Normality Test

<u>Scale Model Constant Variance Test</u>. We test for constant variance by plotting the residuals by the predicted scale values. We visually conclude constant variance if the plotted values are uniformly distributed. Figure 16 displays a reasonably uniform distribution of values for the residual by predicted plot indicating that our model has constant variance.

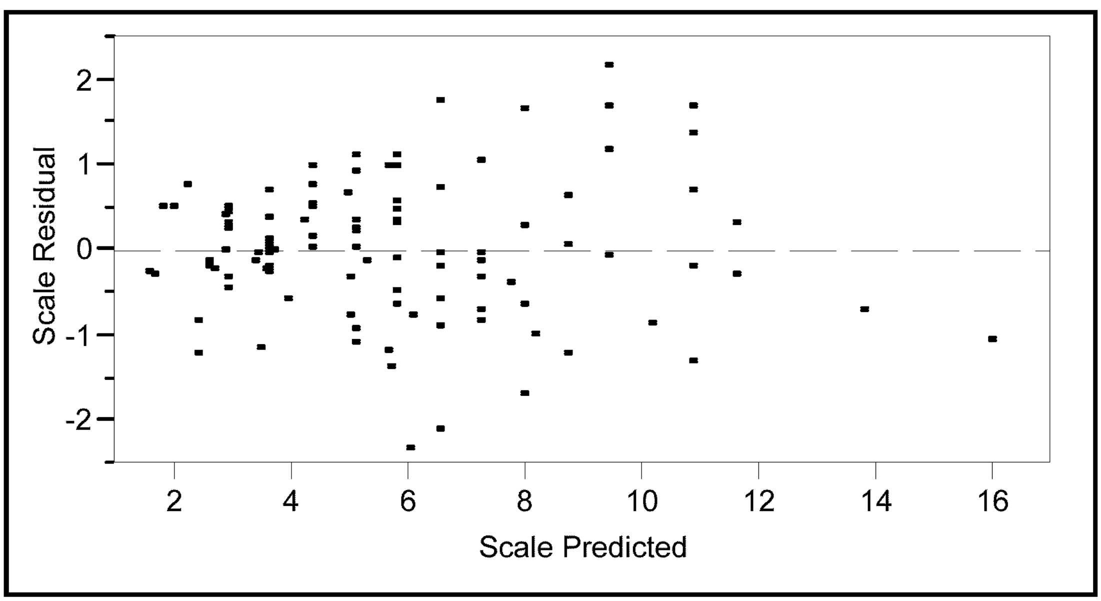

Figure 16. Scale Model Test for Constant Variance

Scale Model Independence of Data Test. The test for independence is not performed. We assume independence in our data because we do not duplicate selected programs. Thus, no obvious serial pattern exists to test for independence.

The Weibull Shape Model. For the shape model results, we display graphically the least squares estimated shape by model predicted shape. Statistically, we present the summary of fit, analysis of variance, and parameter model estimates. Finally, we discuss the shape model as a statistically significant predictor for the Weibull shape parameter.

Figure 17 displays the least squares estimated shape by the model's predicted shape. This figure clearly shows that the least squares estimated shape values are not as correlated with our model's predicted shape in comparison to the scale model. This indicates that our shape regression model does not appear to predict shape as well as our scale model predicts scale.

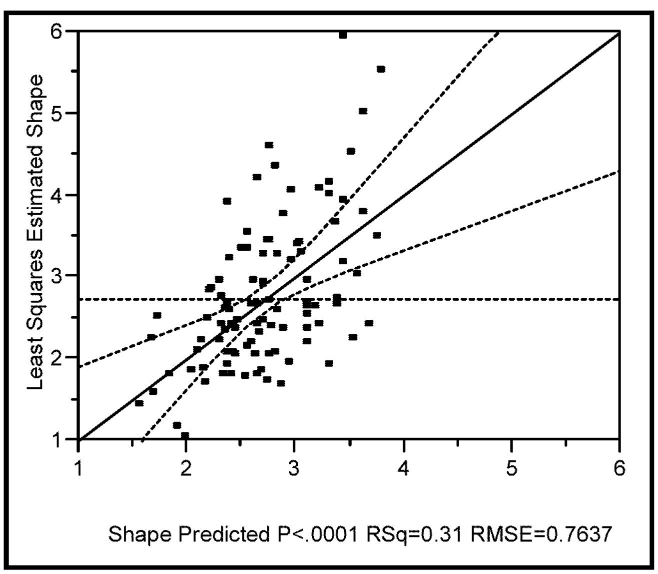

Figure 17. Least Squares Estimated Shape by Predicted Shape Plot

Table 18. Shape Model Table of Statistics

Shape Model Summary of Fit	
RSquare	0.310116
RSquare Adj	**0.274185**
Root Mean Square Error	0.763702
Mean of Response	2.724529
Observations (or Sum Wgts)	102

Shape Model Analysis of Variance (ANOVA)				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	5	25.169127	5.03383	8.6308
Error	96	55.991124	0.58324	**Prob > F**
C. Total	101	81.160251		**<.0001**

Shape Model Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	1.299561	0.32514	4.00	0.0001
ln(1/Duration)	-0.973254	0.16037	-6.07	<.0001
Army	-0.423434	0.20643	-2.05	0.0430
Navy	-0.485661	0.18882	-2.57	0.0116
Electronic	-0.545079	0.18152	-3.00	0.0034
Space	-1.100189	0.56290	-1.95	**0.0536**

Table 18 displays the summary of fit, ANOVA, and parameter model estimates. The adjusted R^2 in the summary of fit table, 0.27, appears to indicate that our model is not a good fit for predicting shape. However, we show later that our shape model predicts the least squares estimated shape well enough to employ the Weibull model with significant results. The overall F test in the ANOVA table shows a *p*-value less than 0.05, indicating that our overall shape model is significant. The t-test in the parameter estimates table shows *p*-values less than 0.05 for all predictors except Space. When the *p*-value is greater than 0.05, the predictor is statistically insignificant. Space is

statistically insignificant but not enough to give up to the additional predictability it adds to the overall shape model. Our shape model does possess one transformation, which implies that there is a better mathematical relationship between ln(1/Duration) and shape than duration simply itself. We define the final shape regression model, β-hat, as

$$\hat{\beta} = \beta_o - \beta_1(\ln(\frac{1}{X_1})) - \beta_2(X_2) - \beta_3(X_3) - \beta_4(X_4) - \beta_5(X_5), \qquad (19)$$

where β_i values are the parameter estimates and X_i are the shape model parameter terms listed in order in Table 18.

To test the statistical soundness of our shape regression model, we look for possible influential data points that bias what explanatory variables are selected, and test the model assumptions of normality, constant variance, and independence.

Shape Model Influential Data Test. Figure 18 displays the overlay plot of values for Cook's D influential statistic, indicating that no observations are significantly influential.

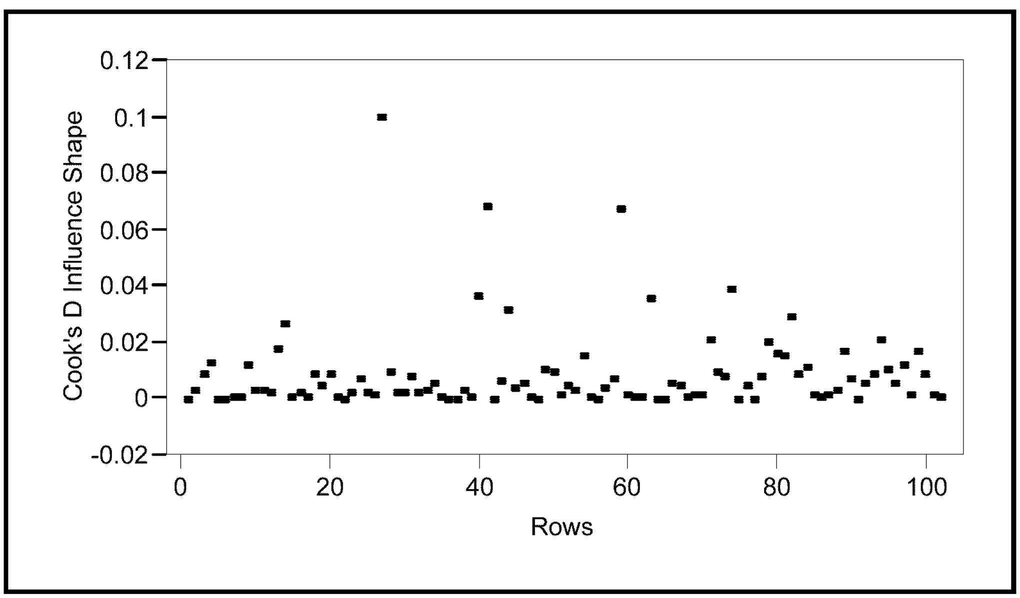

Figure 18. Shape Model Test for Influential Data Points

Shape Model Normality Test. In testing the normality assumption, Figure 19 reveals that the residuals are barely normally distributed, with a p-value of 0.0802.

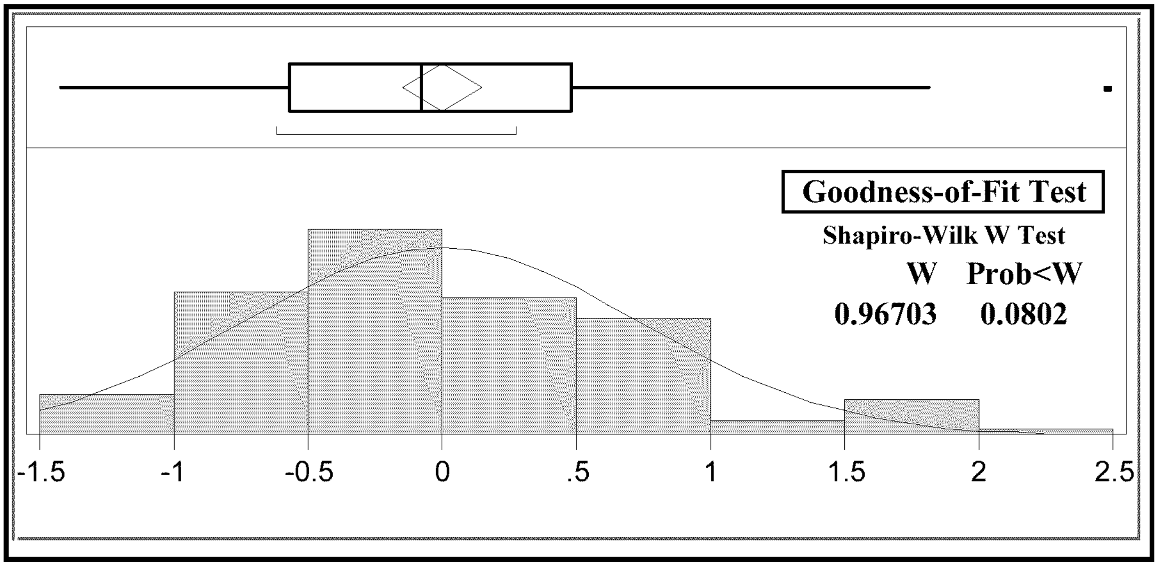

Figure 19. Shape Model Normality Test

However, this is potentially misleading because of one skewed data point displayed in Figure 19 in the box and whiskers plot as a dot.

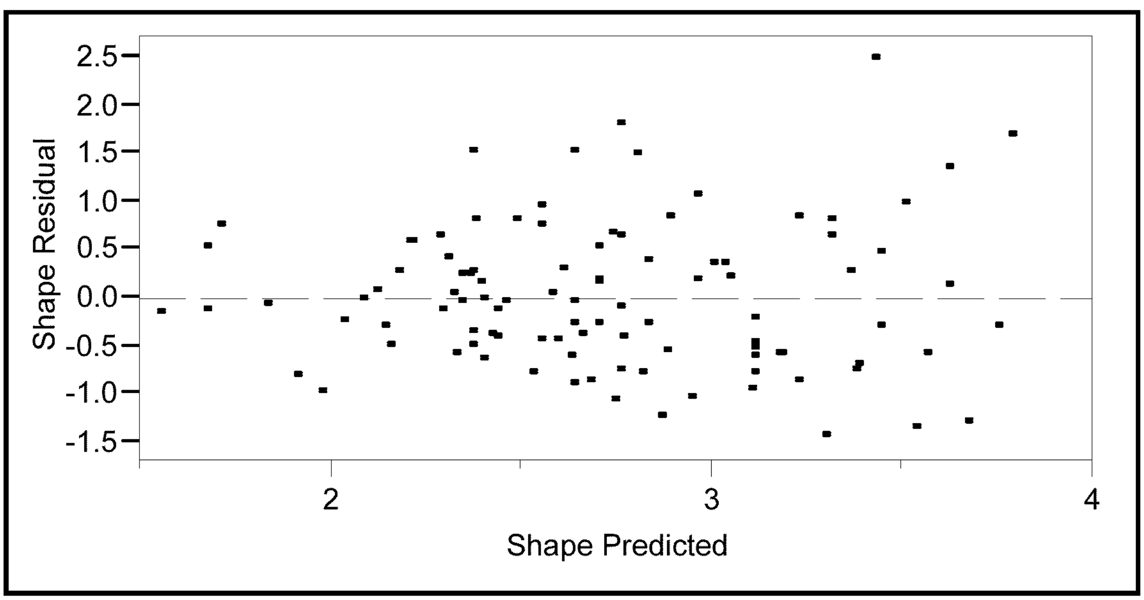

Figure 20. Shape Model Test for Constant Variance

Shape Model Constant Variance Test. Figure 20 shows a reasonably uniform distribution of values for the residual by predicted plot indicating that our shape model has constant variance.

Shape Model Independence of Data Test. The test for independence is not performed. We assume independence in our data because we do not duplicate selected programs. Thus, no obvious serial pattern exists to test for independence.

Scale and Shape Model Validation

We show that both the scale and shape models are statistically significant. The overall F test and the individual parameter t-tests indicate that both overall models and individual predictors within each model report *p*-values less than 0.05. To validate the robustness of our shape and scale regression models, we show that 100 and 96 percent of the remaining 26 (20% of full data set) completed R&D program least squares estimated shape and scale validation data values fall within a 95 percent prediction interval. Our validation results demonstrate that we did not over-fit the data to build the shape and scale regression models. Table 19 and Table 20 display the 95 percent prediction interval upper and lower bounds, the least squares estimated scale and shape values, and the accept/reject results for the scale and shape regression models respectively.

Table 19. Scale Model Validation Using a 95% Prediction Interval

#	Program	LB Scale	Actual Scale	UB Scale	Reject/ Accept
1	ADDS	4.230	5.683	7.517	Accept
2	ALCM	2.776	4.764	6.068	Accept
3	ASM LOSAT	4.230	6.671	7.517	Accept
4	ATACMS	6.402	8.432	9.699	Accept
5	ATACMS-APAM	1.319	2.521	4.623	Accept
6	AVENGER	0.588	2.546	3.902	Accept
7	B-2	6.777	7.323	10.078	Accept
8	BATTLESHIP	2.776	4.017	6.068	Accept
9	C/MH-53E	2.753	3.930	6.044	Accept
10	E-6A	2.228	4.568	5.524	Accept
11	F-14	1.104	2.572	4.411	Accept
12	GLCM	2.776	4.428	6.068	Accept
13	HH-60D	2.048	3.619	5.345	Accept
14	JAVELIN	7.123	6.583	10.428	Reject
15	LANTIRN	3.504	5.631	6.792	Accept
16	LASER HELLFIRE	6.402	8.553	9.699	Accept
17	LBG	2.048	3.532	5.345	Accept
18	MMIII GRP	3.504	4.571	6.792	Accept
19	NAVSTAR GPS	7.844	9.900	11.158	Accept
20	PATRIOT(Missile Seg)	12.148	14.683	15.563	Accept
21	PLSS	7.844	11.017	11.158	Accept
22	ROTHR	3.190	4.337	6.480	Accept
23	RPV	8.564	11.186	11.890	Accept
24	SCAMP	1.319	3.499	4.623	Accept
25	SINCGARS	3.813	5.573	7.100	Accept
26	SSN 21	4.230	7.002	7.517	Accept

Table 20. Shape Model Validation Using a 95% Prediction Interval

#	Program	LB Shape	Actual Shape	UB Shape	Reject/ Accept
1	ADDS	0.787	2.098	3.923	Accept
2	ALCM	1.495	3.322	4.592	Accept
3	ASM LOSAT	1.353	3.718	4.447	Accept
4	ATACMS	1.657	3.512	4.763	Accept
5	ATACMS-APAM	0.670	1.931	3.780	Accept
6	AVENGER	0.378	1.957	3.513	Accept
7	B-2	2.115	2.191	5.241	Accept
8	BATTLESHIP	1.022	1.930	4.094	Accept
9	C/MH-53E	1.016	1.915	4.089	Accept
10	E-6A	0.312	3.289	3.452	Accept
11	F-14	0.540	1.871	3.638	Accept
12	GLCM	1.495	2.678	4.592	Accept
13	HH-60D	1.317	2.521	4.415	Accept
14	JAVELIN	1.738	2.273	4.851	Accept
15	LANTIRN	1.096	2.764	4.201	Accept
16	LASER HELLFIRE	1.657	3.769	4.763	Accept
17	LBG	1.317	2.129	4.415	Accept
18	MMIII GRP	1.644	2.207	4.743	Accept
19	NAVSTAR GPS	1.690	2.429	4.812	Accept
20	PATRIOT (Missile Seg)	2.112	3.845	5.247	Accept
21	PLSS	1.690	4.094	4.812	Accept
22	ROTHR	0.537	2.371	3.664	Accept
23	RPV	1.882	4.043	5.007	Accept
24	SCAMP	-0.785	2.775	3.035	Accept
25	SINCGARS	0.714	2.967	3.851	Accept
26	SSN 21	1.302	4.310	4.374	Accept

The prediction interval calculation does not know to bound shape parameter values to strictly greater than zero. Thus, we note that data point 24, in Table 20, reports an unrealistic negative lower bound shape value.

Now that predictive models for the Weibull shape and scale parameters are established we can employ the Weibull model. In the next section we evaluate the capability of the Weibull model to forecast completed R&D program budget profiles.

Weibull Model Prediction Capability to Actual Budget Profiles

To this point we show that R&D expenditures are Weibull distributed with our GOF results. Our statistical analysis shows that our shape and scale regression models are significant and predict the least squares estimated shape and scale values well. In this section we discuss how we employ Lee, Hogue, and Gallagher's (1997) method to forecast R&D program budget profiles given a point estimate when applying the Weibull model.

We convert completed R&D program budget profiles to Fiscal Year 2000 constant dollar expenditure profiles using (1) and (2). We sum each program's constant dollar expenditure profile to establish a program point estimate, D in CY00$. We convert the total program cost estimate to a Weibull modeled constant dollar expenditure profile with (5) using the regression models to predict the scale and shape with (18) and (19). We convert the constant dollar expenditure profile to current expenditures by multiplying the appropriate raw inflation indices found in Appendix B. Finally, we utilize Lee, Hogue, and Gallagher's (1997) method of constrained nonlinear estimation to develop a budget profile. This method allows the budget estimates, $\hat{B}_i$ for each budget year i, to change simultaneously until an optimal solution is reached. We apply this approach by substituting the $\hat{B}_i$ in (1) and (2) and allow Microsoft Excel Solver function (2000) to select the yearly budget estimates that minimize the sum of squared errors with (14) to

the end of budget completion between the Weibull modeled expenditures and the actual expenditures that result from the forecasted budget profile.

As a note, we minimize the sum of squared errors to the end of budget completion instead of to the end of expenditure completion to produce a more logical program budget profile. This slight change to Lee, Hogue, and Gallagher's (1997) methodology prevents Solver from producing unrealistic spikes at the end of a Weibull model forecasted budget profile. The methodology change does not improve our results to forecast actual budget profiles.

To determine the capability of the Weibull model to forecast budget profiles, we compare the forecasted Weibull-based budget profiles to the 128 completed R&D program budget profiles. We determine the correlation between each completed budget profile and the Weibull forecasted budget profile. To determine the overall Weibull model effectiveness we average the 128 correlation calculations between the completed (Actual) and Weibull (Forecasted) budget profiles. Table 21 summaries our results.

Table 21. Actual by Weibull-Based Budget Profile Correlation Breakdown

Correlation (c) Range	Correlation Distribution	% Correlation Distribution
$c < 0.5$	37	29%
$0.5 \leq c < 0.6$	12	9%
$0.6 \leq c < 0.7$	17	13%
$0.7 \leq c < 0.8$	23	18%
$0.8 \leq c < 0.9$	19	15%
$0.9 \leq c < 1.0$	20	16%
Total	**128**	**100%**
Average Correlation		0.6068
Minimum Correlation		-0.9984
Maximum Correlation		0.9986

We report an average correlation of 0.607 for the 128 programs. This shows that on average 61 percent of the forecasted Weibull-based budget profiles fits the 128 completed program budget profiles.

We find that low correlations between Weibull projected budgets and actual budgets are a result of small program durations. Table 22 displays the percent distribution by budget year duration for programs with correlations less than 0.5. As stated earlier, our GOF results show that only 56 percent of expenditures are Weibull distributed for program durations less than seven years (small discrete distributions). The results show that roughly 59 percent of forecasted Weibull-based budgets are less than 50 percent correlated to completed programs with less than five budget years. Thus, programs with durations less than five years are difficult to forecast budget profiles.

Table 22. Percent Correlation Distribution by Program Duration

Program Duration	Program Distribution	# of Programs w/Correlations Less Than 0.5	% of Programs w/Correlations Less Than 0.5
3	5	3	60%
4	12	7	58%
5	19	6	32%
6	15	6	40%
7	12	3	25%
8	16	4	25%
9	9	4	44%
≥ 10	40	4	10%
Summary			
Duration < 7	51	22	43%
Duration ≥ 7	77	15	19%

Table 23 reports that 4 percent (5 of the 128) of the Weibull modeled budgets are negatively correlated to the actual completed budgets. The negative correlations are

primarily due to small program durations. As an example, Figure 21 displays the Army Avenger combat vehicle final program budget, and the forecasted Weibull-based budgets using the least squares estimates for the shape and scale parameters via Solver, and the predicted shape and scale parameters via our regression models. We show that a negative correlation exists between the actual budget and the Weibull-based budget using the least squares estimates for the shape and scale parameters, indicating that the Avenger expenditure profile is not Weibull distributed. We note, that four of the total five programs that report a negative correlation have less than five budget years.

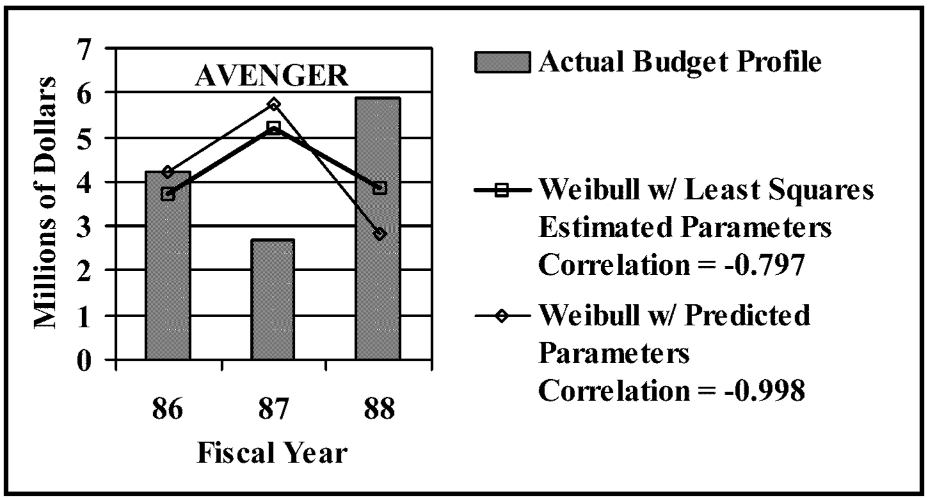

Figure 21. Rayleigh-Based Budget Forecast with Negative Correlation

Weibull and Rayleigh Model Prediction Comparison

Currently, the Rayleigh is the only proven quantitative model we know of that forecasts R&D budget profiles. We define success as being able to improve our ability to forecast completed budget profiles with the Weibull model verses the Rayleigh model. We compare the two models by averaging the calculated correlation between actual and forecasted budget profiles.

We forecast budget profiles for the 128 completed R&D programs using the Rayleigh and Weibull models and compare the calculated average correlation for each model. Table 23 summaries our results and highlights a significant improvement in forecasting budget profiles with the Weibull model as opposed to the Rayleigh. On average, we increase our forecasting ability 60 percent using the Weibull model verses the Rayleigh model. Our findings show that the Weibull model is a better tool to forecast R&D program budget profiles than the current Rayleigh model.

Table 23. Comparing Weibull and Rayleigh-Based Budgets

Correlation (c)	**Correlation Distribution**		**% Correlation Distribution**	
Range	**Rayleigh**	**Weibull**	**Rayleigh**	**Weibull**
$c < 0.0$	67	5	52%	4%
$0.0 \leq c < 0.1$	3	3	2%	2%
$0.1 \leq c < 0.2$	10	3	8%	2%
$0.2 \leq c < 0.3$	6	5	5%	4%
$0.3 \leq c < 0.4$	6	8	5%	6%
$0.4 \leq c < 0.5$	14	13	11%	10%
$0.5 \leq c < 0.6$	5	12	4%	9%
$0.6 \leq c < 0.7$	5	17	4%	13%
$0.7 \leq c < 0.8$	3	23	2%	18%
$0.8 \leq c < 0.9$	3	19	2%	15%
$0.9 \leq c < 1.0$	6	20	5%	16%
Total	**128**	**128**	**100%**	**100%**
Summary				
$c < .5$	106	37	83%	29%
$c \geq .5$	22	91	17%	71%
Correlation Category		**Weibull**	**Rayleigh**	**Delta**
Average Correlation		0.6068	0.0021	0.6047
Minimum Correlation		-0.9984	-0.9051	-0.0934
Maximum Correlation		0.9986	0.9599	0.0387

Our results also show that 83 percent of the Rayleigh modeled budget profiles are less than 50 percent correlated to the actual budget profiles. However, the low average correlation of 0.21 percent between the Rayleigh and actual budget profiles is a result of 52 percent of the Rayleigh modeled budget profiles being negatively correlated with actual budget profiles. Because the Rayleigh model functions with a fixed shape parameter of two, projected budget profiles are over estimate up front and under estimated in the tails (inversely estimated) for 67 of the 128 programs, resulting in negative correlations. As an example, Figure 22 shows the Army Tactical Missile System (ATACMS) completed R&D program budget. The correlation between the ATACMS budget profile and the Weibull and Rayleigh modeled budget profiles is 0.8954 and –0.1798 respectively.

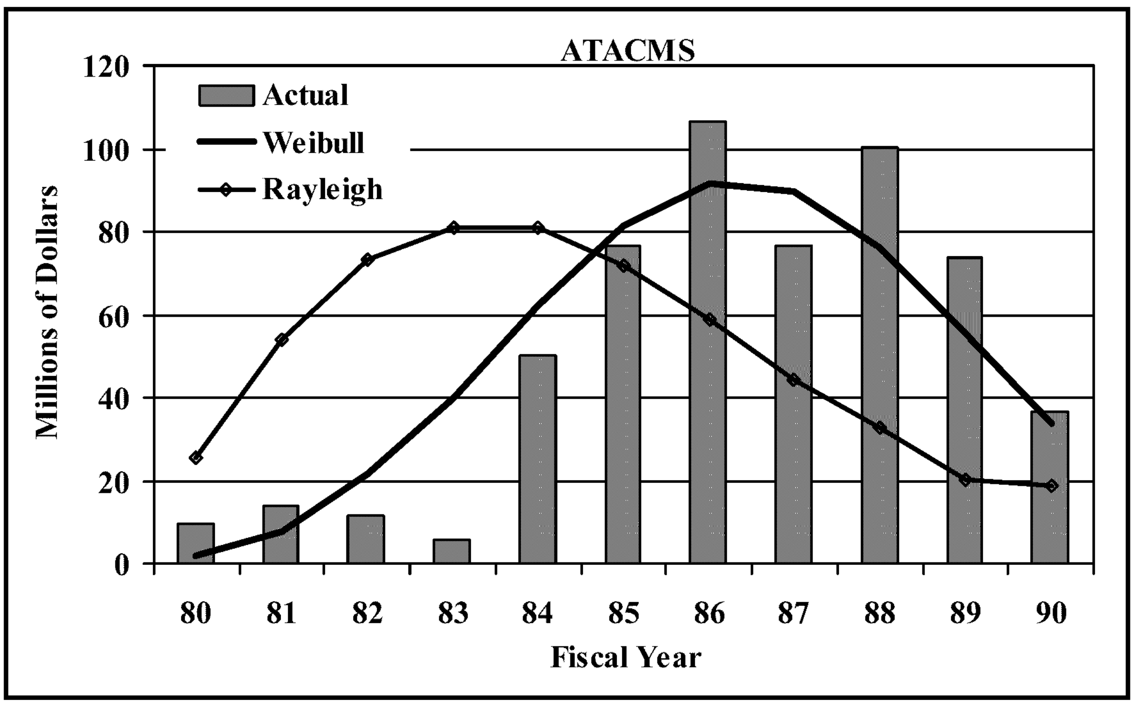

Figure 22. Rayleigh and Weibull Model vs. Actual ATACMS Budget Profile

Chapter Summary

This chapter provides the results of applying the methodology to 128 completed R&D defense program budget profiles. The goodness-of-fit results support our assumption that the Weibull model fits R&D program expenditures. We present the Weibull shape and scale predictive models, and demonstrate that both models are statistically significant. We validate the predictive regression models by showing that 100 and 96 percent of the least squares estimated shape and scale values fall within a 95 percent prediction interval. Using the regression models to predict shape and scale we forecast Weibull-based budgets for 128 completed R&D programs and report an average correlation of 61 percent. Moreover, we define research success as the ability to forecast budget profiles at a higher degree of accuracy using the Weibull as opposed to the Rayleigh model. This chapter reports significant results, showing that the Weibull out performs the Rayleigh model in forecasting 128 completed R&D program budgets on average 60 percent. Chapter Five summarizes and concludes our thesis effort, while addressing possible limitations and future research.

V. Conclusions

Chapter Overview

This chapter presents summary findings and conclusions of this research effort. We include a research summary of Chapters One, Two, and Three, and provide a summary of the results from Chapter Four. Next, we present the limitations to this research effort and provide recommendations for future research. Lastly, comparative conclusions are drawn upon the results of this research effort to previous efforts.

Research Summary

Chapter One introduces the difficultly in forecasting R&D program budget profiles that meet fiscal year expenditure requirements. In addition, it relays that expenditure shortfalls explain over 50 percent of cost overruns and schedule slips. We suggest the Weibull in lieu of the Rayleigh distribution to improve modeling of R&D expenditures. Finally, the chapter concludes with the purpose of determining a mathematical relationship that predicts the necessary shape and scale parameters to forecast Weibull-based program budget profiles.

Chapter Two summarizes previous research in modeling R&D expenditures. We discuss several research efforts that support Rayleigh-based expenditures and budget projections. However, we discuss several theoretical limitations to the Rayleigh and introduce the Weibull model to mitigate those limitations. We present the shape and scale parameters necessary to forecast Weibull-based budget profiles. We conclude the chapter with an example of Lee, Hogue, and Gallagher's (1997) method of forecasting Rayleigh-based budget profiles given a point estimate.

Chapter Three contains our methodology to determine predictive relationships for the Weibull shape and scale parameters. We present the criteria for data selection. We explain the steps to convert R&D budgets to expenditures and our process of nonlinear estimation of the Weibull shape and scale parameters from the converted expenditures. Finally, we present the steps to building our predictive shape and scale models using multiple regression techniques.

Chapter Four presents the results of our methodology applied to 128 completed R&D programs. Using goodness-of-fit statistics we demonstrate that R&D expenditures are Weibull distributed. Using 102 completed R&D defense programs, we develop regression models to predict the necessary Weibull shape and scale parameters. We use the remaining 26 completed R&D programs to validate the robustness of our regression models by showing that 100 and 96 percent of the least squares estimated shape and scale values respectively, fall within our 95 percent prediction intervals. To determine the Weibull model's budget projection capability, we compare 128 completed R&D program budgets to Weibull modeled budgets and report an average correlation of 0.607. To determine the significance of our results we compare the same 128 completed program budgets to Rayleigh modeled budgets. Success is defined as the ability to improve budget profile forecasts with the Weibull as opposed to the Rayleigh model. Using the Weibull over the Rayleigh model when applying Lee, Hogue, and Gallagher's (1997) methodology, we improve initial budget profile projections on average 60 percent.

Limitations

The macro view of this research effort seeks to minimize R&D program cost and schedule growth by developing a better quantitative approach to forecast initial program budget profiles. However, several factors outside of the scope of this research contribute to R&D program cost overrun and schedule slip. Our focus narrows the research scope to funding constraints attributed to budget profiles that do not meet fiscal expenditure requirements. Within this scope of focus we find other limitations to forecasting Weibull-based budget profiles. First, R&D programs with four or less budget years rarely execute expenditures that follow any distribution consistently. Therefore, forecasted Weibull-based budgets on average show poor correlations to final programs with four or less budget years. Secondly, our research only applies to Army, Navy, and Air Force Acquisition Category One (ACAT I) R&D programs. This is due to our data source. The Department of Defense only requires system program offices to submit a Selected Acquisition Report (SAR) for ACAT I programs.

Future Research

We show in this research that the Weibull out performs the Rayleigh model in projecting R&D program budget profiles. Recommendations for future research include: a determination whether the Weibull model out performs current practices in forecasting initial R&D budgets, and applying the research methodology to other existing data bases that include lower ACAT R&D programs. We could not explore performance comparisons between historical initial forecasted R&D program budgets and Weibull-modeled budgets based on initial program cost and schedule estimates to the final

program budgets, because only 13 of the 128 programs had initial budget profile estimates. This number of programs is too low to draw any statistical conclusions.

Conclusion

We conclude by stating that our proposed methodology provides the requisite shape and scale regression models to forecast Weibull-based R&D budgets. Currently, the Rayleigh model serves as the only quantitative tool to project budget profiles. The Weibull out performs the Rayleigh in forecasting 128 completed R&D budgets on average 60 percent. We consider an average increase of 60 percent in budget profile prediction accuracy excellent results. Thus, we achieve our research objective and provide an improved R&D budget-forecasting tool.

Appendix A. Program Regression Model Building Data

Table 24. Program Model Building Data

#	Programs	Lead Service	System Type	Dur - γ	Dur	Locn γ	Scale δ	Shape β
1.	A-6E/F	N	Aircraft	5.000	5	0.000	3.811	2.649
2.	ABRAMS UPGRADE	A	Vehicle	10.000	10	0.000	6.501	2.598
3.	ACM	AF	Missile	3.582	5	1.418	2.483	1.755
4.	ADDS	A	Electronic	8.000	8	0.000	5.683	2.098
5.	AFATDS	A	Electronic	15.000	15	0.000	12.314	3.163
6.	ALCM	AF	Missile	6.000	6	0.000	4.764	3.322
7.	AMRAAM	AF	Missile	11.000	11	0.000	8.341	3.754
8.	AN/BSY-1	N	Electronic	9.000	9	0.000	4.501	1.776
9.	AN/BSY-2	N	Electronic	16.000	16	0.000	11.376	4.025
10.	AN/SQQ-89	N	Electronic	8.000	8	0.000	6.450	2.929
11.	AN/TTC-39	A	Electronic	9.000	9	0.000	6.419	2.423
12.	AOE 6	N	Ship	7.000	7	0.000	4.050	2.436
13.	APACHE (AH-64)	A	Aircraft	10.000	10	0.000	7.194	2.665
14.	ASAS	A	Electronic	8.000	8	0.000	5.776	2.590
15.	ASAT	AF	Missile	13.000	13	0.000	11.660	5.487
16.	ASM LOSAT	A	Vehicle	8.000	8	0.000	6.671	3.718
17.	ASPJ	N	Electronic	11.002	12	0.998	6.361	2.154
18.	ATACMS	A	Missile	11.000	11	0.000	8.432	3.512
19.	ATACMS-APAM	A	Missile	4.000	4	0.000	2.521	1.931
20.	ATARS	AF	Electronic	6.914	9	2.086	4.308	2.022
21.	AV-8B	N	Aircraft	8.000	8	0.000	6.201	3.233
22.	AVENGER	A	Vehicle	3.000	3	0.000	2.546	1.957
23.	B-1 CMUP-JDAM	AF	Electronic	3.941	5	1.059	2.926	2.070
24.	B-1B	AF	Aircraft	4.000	4	0.000	3.471	2.622
25.	B-2	AF	Aircraft	11.520	13	1.480	7.323	2.191
26.	B-52 OAS/CMI	AF	Electronic	4.980	5	0.020	4.068	2.735
27.	BATTLESHIP	N	Ship	6.000	6	0.000	4.017	1.930
28.	BFVS	A	Vehicle	15.000	15	0.000	12.631	4.498
29.	BFVS UPGRADE	A	Vehicle	7.000	7	0.000	4.221	2.010
30.	BLACKHAWK	A	Aircraft	6.818	9	2.182	5.675	3.421
31.	C/MH-53E	N	Aircraft	5.967	6	0.033	3.930	1.915
32.	C-17A	AF	Aircraft	9.000	9	0.000	8.366	5.922
33.	CAPTOR	N	Munition	12.000	12	0.000	9.420	4.063
34.	CH-47D	A	Aircraft	4.937	5	0.063	3.440	2.041

#	Programs	Lead Service	System Type	Dur - γ	Dur	Locn γ	Scale δ	Shape β
35.	CH-60S	N	Aircraft	4.955	5	0.045	3.690	2.610
36.	CMU	AF	Electronic	22.000	22	0.000	14.976	3.458
37.	COPPERHEAD	A	Munition	9.000	9	0.000	7.319	3.366
38.	CSRL	AF	Munition	4.000	4	0.000	3.298	2.390
39.	CV HELO	N	Aircraft	4.000	4	0.000	2.512	1.676
40.	CVN 68	N	Ship	4.730	5	0.270	3.467	2.383
41.	DDG-51	N	Ship	7.000	7	0.000	5.389	2.863
42.	DSCS III	AF	Space	5.396	6	0.604	3.400	1.771
43.	E-3A	AF	Aircraft	11.000	11	0.000	9.701	4.993
44.	E-6A	N	Electronic	5.247	6	0.753	4.568	3.289
45.	EF-111A	AF	Aircraft	6.913	7	0.087	4.772	2.606
46.	EJS	AF	Electronic	6.000	6	0.000	5.190	3.324
47.	F/A-18	N	Aircraft	5.095	6	0.905	3.768	2.549
48.	F/A-18 E/F	N	Aircraft	8.308	9	0.692	3.783	1.642
49.	F-14	N	Aircraft	3.706	4	0.294	2.572	1.871
50.	F-14D	N	Aircraft	4.621	6	1.379	3.306	2.182
51.	F-16	AF	Aircraft	2.489	3	0.511	2.381	2.457
52.	FAAD C2I	A	Electronic	8.000	8	0.000	6.213	2.302
53.	FAADS NLOS	A	Missile	5.000	5	0.000	3.452	2.332
54.	FDS	N	Electronic	13.000	13	0.000	9.444	3.408
55.	FFG 7	N	Ship	2.156	3	0.844	1.367	1.405
56.	FMTV	A	Vehicle	3.690	4	0.310	2.515	1.855
57.	GLCM	AF	Missile	6.000	6	0.000	4.428	2.678
58.	HARM	N	Missile	10.000	10	0.000	8.386	3.266
59.	HARPOON	N	Missile	6.000	6	0.000	4.970	3.312
60.	HH-60D	AF	Aircraft	5.000	5	0.000	3.619	2.521
61.	I-S/A AMPE	AF	Electronic	3.318	4	0.682	1.632	1.125
62.	IUS	AF	Space	7.247	8	0.753	5.196	2.196
63.	JAVELIN	A	Missile	12.000	12	0.000	6.583	2.273
64.	JDAM	AF	Munition	7.879	8	0.121	4.401	1.891
65.	JSIPS	AF	Electronic	8.000	8	0.000	5.383	2.372
66.	JSOW	N	Missile	15.000	15	0.000	9.642	3.911
67.	JSTARS	AF	Electronic	15.000	15	0.000	10.756	2.693
68.	JTUAV	N	Aircraft	7.000	7	0.000	6.263	3.237
69.	KC-135R	AF	Aircraft	6.000	6	0.000	5.416	3.396
70.	KIOWA	A	Aircraft	6.000	6	0.000	4.457	2.914
71.	LAMPS	N	Electronic	13.000	13	0.000	11.186	4.574

#	Programs	Lead Service	System Type	Dur - γ	Dur	Locn γ	Scale δ	Shape β
72.	LANTIRN	AF	Electronic	7.000	7	0.000	5.631	2.764
73.	LASER HELLFIRE	A	Missile	11.000	11	0.000	8.553	3.769
74.	LBG	AF	Munition	5.000	5	0.000	3.532	2.129
75.	LCAC	N	Ship	5.000	5	0.000	4.392	3.889
76.	LHD 1	N	Ship	3.321	5	1.679	1.275	1.014
77.	LONGBOW APACHE	A	Electronic	11.000	11	0.000	7.405	2.296
78.	LONGBOW HELLFIRE	A	Missile	5.000	5	0.000	3.429	2.022
79.	LSD 41	N	Ship	5.000	5	0.000	3.509	2.033
80.	LSD 41 CV	N	Ship	6.000	6	0.000	4.940	3.514
81.	MAVERICK	AF	Missile	8.000	8	0.000	6.357	3.975
82.	MCM 1	N	Ship	5.000	5	0.000	3.769	2.031
83.	MCS	A	Electronic	4.000	4	0.000	3.239	2.205
84.	MHC 51	N	Ship	3.546	4	0.454	2.520	1.815
85.	MK 48 ADCAP	N	Missile	7.000	7	0.000	5.484	2.894
86.	MK 50 TORPEDO	N	Missile	15.000	15	0.000	11.668	3.148
87.	MLRS	A	Munition	3.947	5	1.053	3.335	2.799
88.	MLRS-TGW	A	Munition	13.000	13	0.000	10.673	3.634
89.	MLS	AF	Electronic	7.000	7	0.000	6.077	4.176
90.	MMIII GRP	AF	Missile	7.000	7	0.000	4.571	2.207
91.	MMIII PRP	AF	Missile	7.000	7	0.000	5.172	2.614
92.	NAS	AF	Electronic	9.000	9	0.000	6.557	2.330
93.	NAVSTAR GPS	AF	Electronic	13.000	13	0.000	9.900	2.429
94.	NESP	N	Electronic	12.000	12	0.000	7.554	1.816
95.	OTH-B	AF	Electronic	2.698	3	0.302	2.550	2.487
96.	PATRIOT	A	Missile	16.000	16	0.000	11.990	3.005
97.	PATRIOT(Fire Unit)	A	Missile	10.000	10	0.000	7.015	2.327
98.	PATRIOT(Missile Seg)	N	Missile	19.000	19	0.000	14.683	3.845
99.	PEACEKEEPER	AF	Missile	4.000	4	0.000	2.644	1.768
100.	PERSHING II	A	Missile	8.000	8	0.000	6.985	3.739
101.	PHOENIX	N	Missile	8.000	8	0.000	5.227	2.565
102.	PLS	A	Vehicle	2.295	3	0.705	1.453	1.560
103.	PLSS	AF	Electronic	13.000	13	0.000	11.017	4.094
104.	ROTHR	N	Electronic	6.569	7	0.431	4.337	2.371
105.	RPV	A	Aircraft	14.000	14	0.000	11.186	4.043
106.	RSIP	AF	Electronic	7.787	8	0.213	4.540	1.694
107.	SADARM	A	Munition	10.000	10	0.000	7.296	2.514
108.	SCAMP	A	Space	4.000	4	0.000	3.499	2.775

#	Programs	Lead Service	System Type	Dur - γ	Dur	Locn γ	Scale δ	Shape β
109.	SFW	AF	Munition	10.000	10	0.000	6.615	2.202
110.	SGT YORK GUN	A	Munition	5.821	6	0.179	4.637	2.627
111.	SH-60R	N	Aircraft	14.000	14	0.000	9.365	2.620
112.	SINCGARS	A	Electronic	7.426	8	0.574	5.573	2.967
113.	SLAT	N	Missile	12.000	12	0.000	8.832	2.379
114.	SPARROW	N	Missile	6.000	6	0.000	4.593	2.127
115.	SRAM II	AF	Missile	8.000	8	0.000	6.866	4.123
116.	SSN 21	N	Ship	8.000	8	0.000	7.002	4.310
117.	STINGER	A	Missile	7.000	7	0.000	5.367	2.684
118.	T-45TS	N	Aircraft	10.666	13	2.334	7.428	2.917
119.	T-46A	AF	Aircraft	3.056	5	1.944	3.059	3.185
120.	TACTAS	N	Electronic	9.000	9	0.000	6.024	2.389
121.	T-AGOS	N	Ship	5.000	5	0.000	3.678	1.894
122.	TOMAHAWK	N	Missile	8.997	10	1.003	5.694	1.916
123.	TOW 2	A	Missile	4.000	4	0.000	3.419	2.822
124.	TRIDENT II MSL	N	Missile	7.780	10	2.220	6.689	4.314
125.	TRI-TAC	AF	Electronic	8.395	10	1.605	5.391	2.043
126.	UGM-84	N	Missile	4.789	5	0.211	2.404	1.765
127.	V-22	N	Aircraft	19.000	19	0.000	13.165	2.379
128.	WISWAM	AF	Electronic	11.278	12	0.722	7.270	2.152

Appendix B. Service Raw Inflation Indices

Table 25. RDT&E FY2000 Raw Inflation Indices

FY	Air Force	Army	Navy	FY	Air Force	Army	Navy
1965	0.2100	0.2123	0.2163	1993	0.9011	0.9038	0.9038
1966	0.2156	0.2194	0.2221	1994	0.9192	0.9219	0.9219
1967	0.2225	0.2273	0.2293	1995	0.9366	0.9394	0.9394
1968	0.2305	0.2350	0.2376	1996	0.9554	0.9582	0.9582
1969	0.2414	0.2424	0.2488	1997	0.9754	0.9754	0.9754
1970	0.2546	0.2577	0.2625	1998	0.9822	0.9822	0.9822
1971	0.2676	0.2693	0.2760	1999	0.9901	0.9901	0.9901
1972	0.2799	0.2796	0.2887	2000	1.0000	1.0000	1.0000
1973	0.2923	0.2910	0.3012	2001	1.0150	1.0150	1.0150
1974	0.3151	0.3082	0.3253	2002	1.0302	1.0302	1.0302
1975	0.3491	0.3547	0.3608	2003	1.0457	1.0457	1.0457
1976	0.3732	0.3750	0.3847	2004	1.0666	1.0666	1.0666
1977	0.3985	0.4057	0.4060	2005	1.0879	1.0879	1.0879
1978	0.4256	0.4346	0.4336	2006	1.1097	1.1097	1.1097
1979	0.4614	0.4741	0.4700	2007	1.1319	1.1319	1.1319
1980	0.5048	0.5243	0.5198	2008	1.1545	1.1545	1.1545
1981	0.5648	0.5799	0.5749	2009	1.1776	1.1776	1.1776
1982	0.6168	0.6240	0.6186	2010	1.2012	1.2012	1.2012
1983	0.6470	0.6489	0.6489	2011	1.2252	1.2252	1.2252
1984	0.6716	0.6736	0.6736	2012	1.2497	1.2497	1.2497
1985	0.6944	0.6964	0.6965	2013	1.2747	1.2747	1.2747
1986	0.7139	0.7160	0.7160	2014	1.3002	1.3002	1.3002
1987	0.7332	0.7353	0.7353	2015	1.3262	1.3262	1.3262
1988	0.7552	0.7574	0.7574	2016	1.3527	1.3527	1.3527
1989	0.7869	0.7892	0.7892	2017	1.3798	1.3798	1.3798
1990	0.8184	0.8216	0.8208	2018	1.4073	1.4073	1.4073
1991	0.8535	0.8569	0.8561	2019	1.4355	1.4355	1.4355
1992	0.8774	0.8826	0.8800				

Bibliography

Abernethy, T., An Application of the Rayleigh Distribution to Contract Cost Data. MS Thesis, Naval Postgraduate School, Monterey CA, 1984. (AD-A151551)

Belcher, Gerald J. and John A. Dukovich, "Improved Methods for Estimating Development Costs" Logistics Management Institute, 1999.

Gallagher, Mark A. and David A. Lee, "Final Cost Estimates for Research and Development Programs Conditioned on Realized Costs" Military Operations Research Journal: Vol. 2, No 2, pp. 51-65 (Summer 1996).

Hines, William W. and Douglas C. Montgomery. Probability and Statistics in Engineering and Management Science (2nd Edition), pp. 164-165. New York: John Wiley & Sons, 1980.

Jarvis, Will and Ed Pohl. Lecture Slides, Program Execution Efficiency and Resource Phasing. Department of Defense Cost Analysis Symposium (DODCAS), Washington DC, November 1999.

Lee, David A., M. R. Hogue, and Mark A. Gallagher, "Determining a Budget Profile from a R&D Cost Estimate" Journal of Cost Analysis: 29-41 (Fall 1997).

Microsoft Office 2000 Professional, Microsoft Corporation, Seattle WA, 2000.

Neter, John, Michael H. Kutner, Christopher J. Nachtsheim, and William Wassweman, Applied Liner Statistical Models (Fourth Edition), pp. 380-384. Boston: MaGraw-Hill Companies, Inc., 1996.

Norden, P. V., "Useful Tools for Project Management," Management of Production, Reprinted from Operations Research in Research and Development. Edited by H.V. Dean., John Wiley & Sons, Inc., 1970, pp. 113-143.

Outlay Rates to be used for incremental Changes in BA Purchases (Table 4). National Defense Budget Estimates for FY2000 (The Greenbook), office of the Undersecretary of Defense (Comptroller). Department of Defense, Washington DC.

Porter, Paul H., Revising R&D Program Budgets When Considering Funding Curtailment with a Weibull Model. MS Thesis, Air Force Institute of Technology, Wright-Patterson AFB OH, 2001. (AD-A391273)

Putnam, Lawrence H., "A General Empirical Solution to the Macro Software Sizing and Estimation Problem," IEEE Transactions on Software Engineering, Vol. SE-4, NO. 4, July 1978, pp. 345-361.

Sall, John, Ann Lehman, and Lee Creighton, JMP Start Statistics; "A Guide to Statistics and Data Analysis Using JMP® and JMP IN® Software." SAS Institute Inc., 2001.

Unger, Eric J., Relating Initial Budget to Program Growth with Rayleigh and Weibull Models. MS Thesis, Air Force Institute of Technology, Wright-Patterson AFB OH, 2001. (AD-A390987)

Watkins, Harry, An Application of Rayleigh Curve Theory to Contract Cost Estimation and Control. MS Thesis, Naval Postgraduate School, Monterey CA, 1982. (AD-A118213)

Vita

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13. SUPPLEMENTARY NOTES

14. ABSTRACT

Norden (1970) uses the Rayleigh, which is a degenerative of the Weibull, to model manpower on research and development (R&D) programs. Several research efforts extend his work including Lee, Hogue, and Gallagher (1997) who build R&D program budgets based on Rayleigh expenditures. We demonstrate the theoretical limitations to the Rayleigh model and present the Weibull model, which mitigates those limitations. Using 102 completed R&D defense programs, we develop regression models to predict the requisite shape and scale parameters to forecast Weibull-based budgets. Using the remaining 26 completed R&D programs to validate the robustness of our regression models, we show that 100 and 96 percent of the least squares estimated shape and scale values respectively, fall within a 95 percent prediction interval. We determine the Weibull model's budget projection capability by comparing forecasted Weibull-based budgets to 128 completed R&D program budgets and report an average correlation of 0.607. To determine the significance of our results we compare forecasted Rayleigh-based budgets to the same 128 completed program budgets. Using the Weibull over the Rayleigh model when applying Lee, Hogue, and Gallagher's (1997) methodology, we improve initial budget profile projections on average 60 percent.

15. SUBJECT TERMS

Cost Models, Cost Analysis, Budget Estimates, Cost Estimates, Military Budgets

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